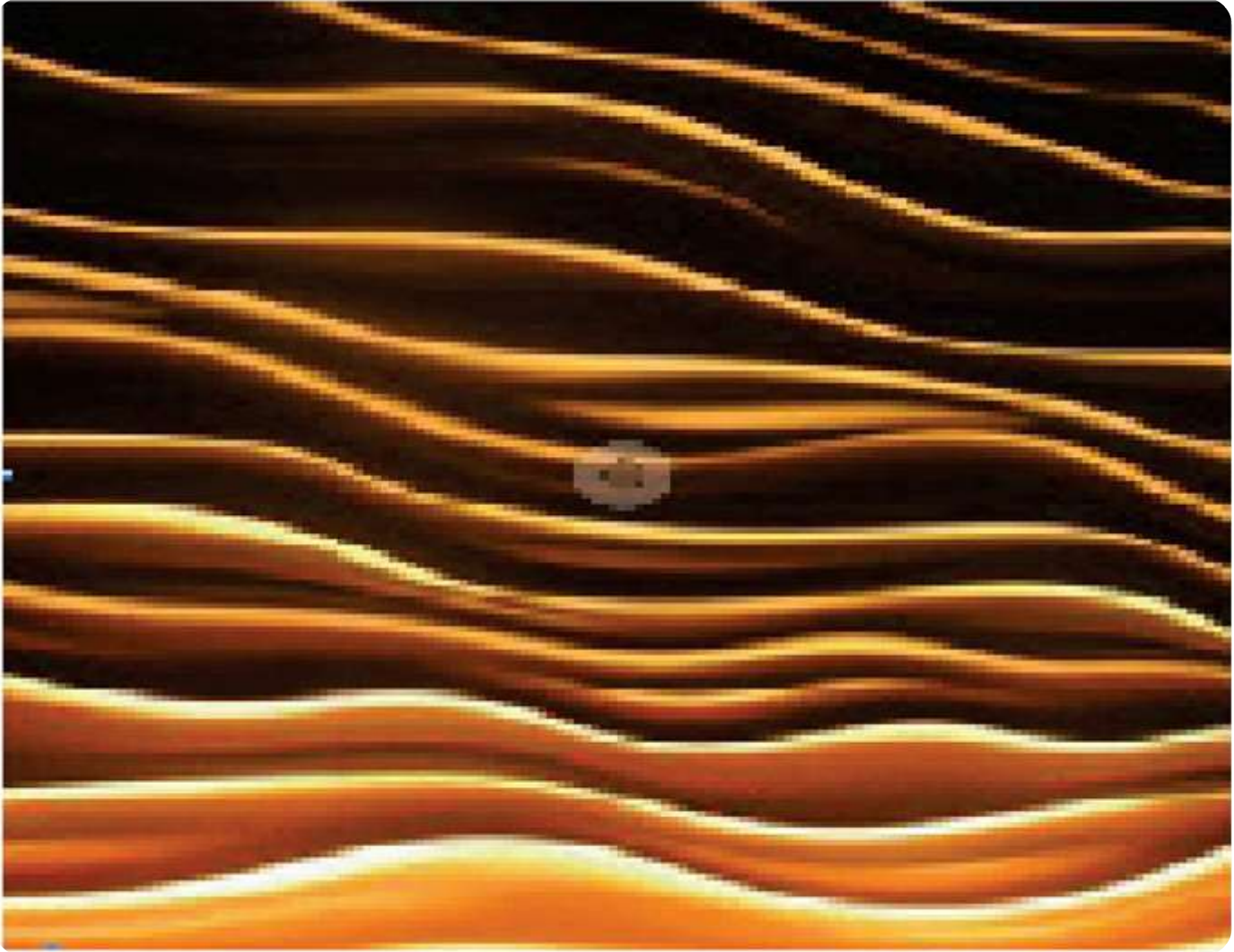




Key unit competence: By the end of the unit I should be able to evaluate the propagation of mechanical waves.

Unit Objectives:

By the end of this unit I will be able to;

- ◇ Explain the terms, concept and characteristics of waves properly.
- ◇ Explain the properties of waves.
- ◇ Explain the behavior of waves in vibrating strings and applications

?

of waves properly.

Introductory Activity

- a. Arrange yourselves the form of a circle with your right shoulders pointing towards the centre.
- b. Ask your friend to raise arms and then lower them. Then the next friend raises arms and lowers them, and so on around the circle. It should be like the "wave".
- c. Describe the type of the disturbance formed.
- d. Is the disturbance travelling up and down or horizontally around the circle?
- e. Let one of your friend gently push the back of the next student and then the pushed member should gently push the next member and so on, which will make a wave travel around the ring.
- f. From what you have done, can you describe what a disturbance is? Is the disturbance travelling up and down or around the ring?

4.0 INTRODUCTION

When we think of the word "wave", we usually visualize someone moving his hand back and forth to say 'hello' or maybe we think of a tall curling wall of water moving in from the ocean to crash on the beach.

In physics, a wave is a disturbance that occurs in a material medium and in such process, energy is transferred from one place to another. When studying waves, it's important to remember that they transfer energy, not matter.

There are lots of waves all around us in everyday life. Sound is a type of wave that moves through matter and then vibrates our eardrums and we hear. Light is a special kind of wave that is made up of photons that helps us to see. You can drop a rock into a pond and see wave formation in the water. We even use waves (microwaves) to cook our food really fast. Application of this concept is extensively used in telecommunication and music.

4.1 THE CONCEPT OF WAVES

Waves can be defined as a disturbance in a medium that transfers energy from one place to another, although the medium itself does not travel.

The term wave is often intuitively understood as referring to a transport

of spatial disturbances that are generally not accompanied by a motion of the medium occupying this space as a whole. In a wave, the energy of a vibration is moving away from the source in the form of a disturbance within the surrounding medium. Other properties, however, although

usually described in terms of origin, may be generalized to all waves. For such reasons, wave theory represents a particular branch of physics that is concerned with the properties of wave processes independently of their physical origin.

4.2 TERMS USED AND CHARACTERISTICS OF WAVES

All waves are characterized by the following terms;

The **Time period (T)** of the wave is the time it takes for one wavelength of the wave to pass a point in space or the time for one cycle to occur. It is also defined as the time taken between two successive wave crests or trough. It is measured in seconds (s).

The **frequency (f)** is the number of wavelengths that pass a point in space per second. In another words, it can be defined as the number of complete oscillations or vibrations per second. Its SI unit is hertz (Hz). Mathematically;

$$f = \frac{1}{T}$$

..... Equation 4-1

The wavelength (λ) is the horizontal distance in space between two nearest points that are oscillating in phase (in step) or the spatial distance over which the wave makes one complete oscillation. Its SI unit is metre (m).

The **wave speed (v)** is the speed at which the wave advances. Its SI unit is m/s.

$$\text{Speed} = \frac{\text{distance travelled}}{\text{time period}} = \frac{\lambda}{T} \quad \text{..... Equation 4-2}$$

$$v = \lambda f$$

That is, wave speed = wavelength × frequency.

This is the relationship between wavelength, frequency and velocity.

Amplitude is defined as the maximum distance measured from equilibrium position (mean position). The amplitude is always taken as positive and is measured in metres.

Phase difference (phase angle) is the angular difference between two points on the wave or between two waves. Consider, two points O and P on the wave as shown in Fig. 4-12.

Phase difference is a whole number and is calculated using simple proportions;

$$\phi = \frac{2\pi x}{\lambda}$$

..... Equation 4-3

The **wave number**, also called the propagation number k , is the spatial frequency of a wave, either in cycles per unit distance or radians per unit distance. It can be envisaged as the number of waves that exist over a specified distance (analogous to frequency being the number of cycles or radians per unit time). Its unit is per metre (m^{-1}). Mathematically;

$$k = \frac{\Phi}{x} = \frac{2\pi}{\lambda}$$

..... Equation 4-4

The **Intensity (I)** of a wave or the power radiated by a source are proportional to the square of the amplitude (x).

$$I \propto x^2$$

Wavefront is a line or surface in the path of the wave motion on which the disturbance at every point have the same phase. This can also be defined as the surface which touches all the wavelets from the secondary sources of waves. Consider the Huygens construction principle for the new wavefront.

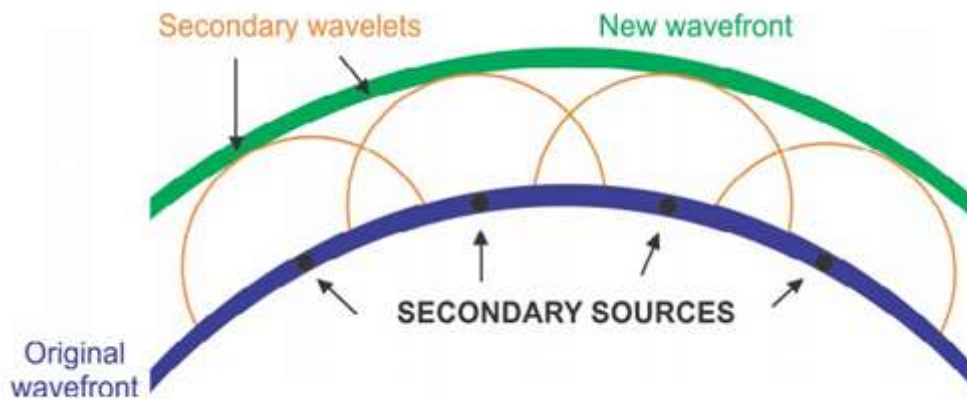


Fig. 4.1: Formation of the wavefront

Crest is the highest point above the equilibrium position while trough is the lowest point below then equilibrium position.

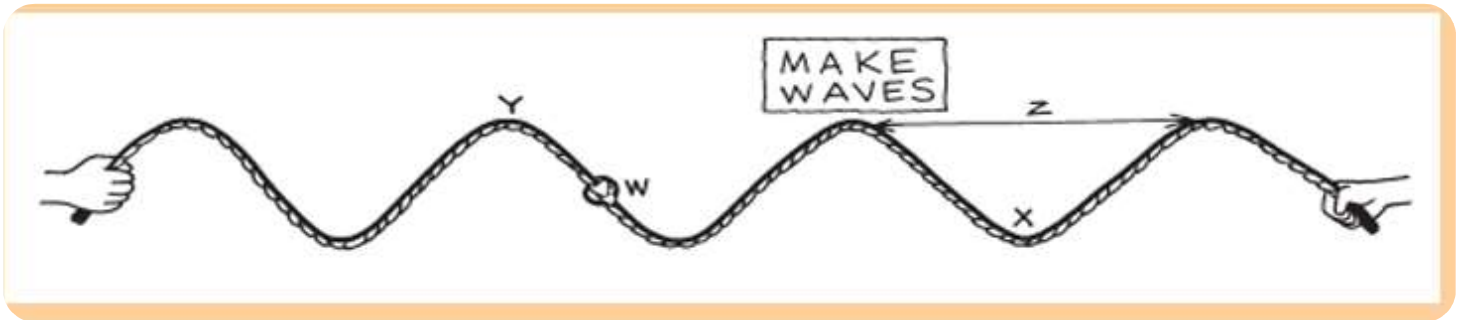
The angular frequency ω represents the frequency in radians per second. It is related to the frequency by

$$\omega = 2\pi f = \frac{2\pi}{T}$$

A **node** is a point half way between the crest and the trough. The line that connects the nodes is the **nodal line**. The nodal line shows the original position of the matter carrying the wave.

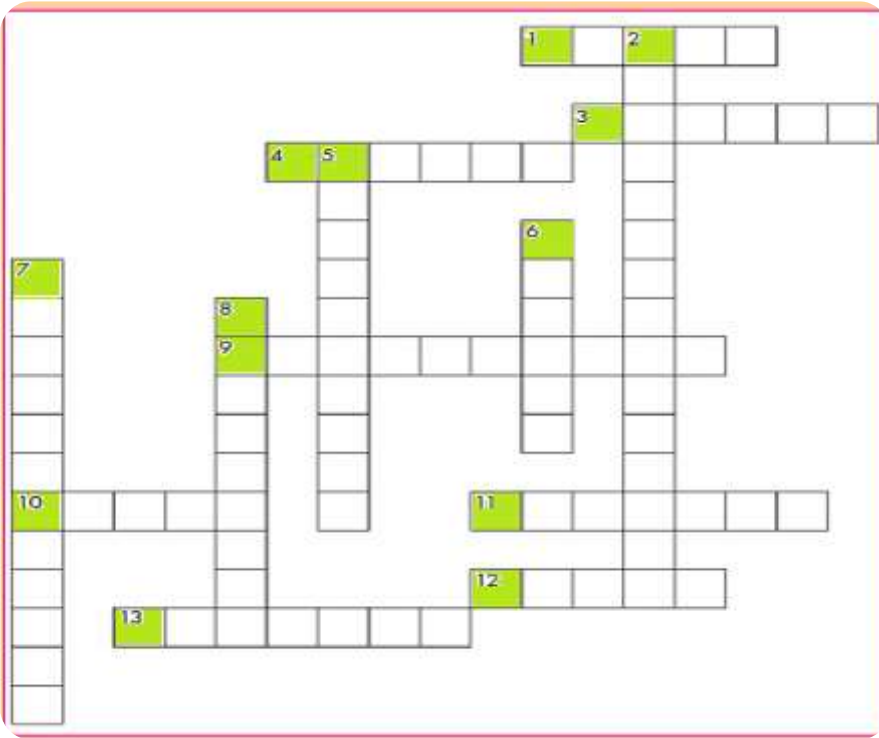
Application Activity 4.1

1. Requirements: a manila paper with the drawing of the wave shown below



- How do you call the distance represented by arrow z?
- What letter is labelling the wave's trough?
- What letter is labelling a wave's crest?
- The number of waves that pass the poster per second is called the of the waves.
- If the knot (w) travels 2 meters in 1 second, we say that it has of 2 m/s.
- If the wavelengths were shortened, would the frequency be higher or lower?
- The greatest distance the knot (w) travels from its resting position is called..... of the wave.
- What kind of wave are these in the rope?

2. Use the following descriptions in waves ad fill in the crossword puzzle bellow:



Across

1. How fast something is moving or how much distance is covered in a certain amount of time.
3. The time it takes for a wave to repeat itself
4. The lowest point of a wave beneath the line of origin
9. Waves that require a medium
10. The highest point of a wave above the line of origin
11. Particles of light
12. A push or a pull
13. The tendency of an object at rest to remain at rest or in motion until acted upon

Down

1. Waves that do not require a medium
2. The bouncing back of a wave when it meets the surface or boundary
3. The matter through which a wave travels
4. Distance in a given direction
5. The vertical distance between the line of origin and the crest of a wave

4.3 TYPES OF WAVES

Waves are of three main types: Mechanical wave, electromagnetic wave and matter wave.

These waves are classified based on conditions necessary for the wave to propagate

4.3.1 Mechanical waves

These waves are produced by the disturbance in a material medium and they are transferred by particles of the medium.

The matter through which mechanical waves travel is called the medium.

All mechanical waves require (1) some source of disturbance, (2) a medium that can be disturbed, and (3) some physical mechanism through which elements of the medium can influence each other.

Mechanical Waves are divided into two types according to the direction of the displacements in relation to the direction of the motion of the wave itself (wave form):

a) Longitudinal waves

When a wave propagates through some medium and the local displacements of the medium that constitute the disturbance are in the direction of travel of the disturbance, then the wave is longitudinal.

An example of a longitudinal wave is the pulse that can be sent along a stretched slinky by shaking one end of the slinky along its length. The pulse moves along the line of the slinky and ultimately makes the other end move. Notice that in this case, the individual coils of the slinky vibrate back and forth about some equilibrium position, but there is no net movement of the slinky itself.

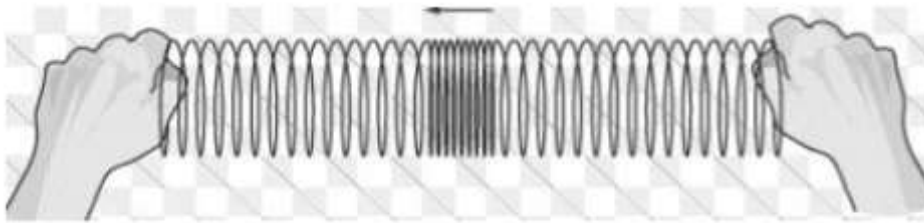


Fig. 4.2: Longitudinal wave

b) Transverse waves

These are waves in which the direction of disturbance is perpendicular to the direction of travel of the wave. The particles do not move along with the wave; they simply oscillate up and down about their individual equilibrium positions as the wave passes by.

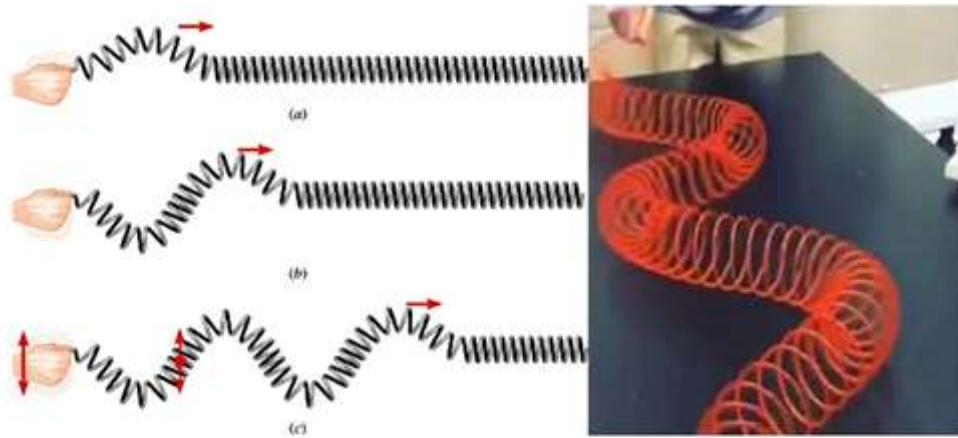


Fig. 4.3: Transverse waves

4.3.1.4 Examples of mechanical waves

Mechanical waves, being progressive and stationary, are seen in different forms as described in this section.

Sound waves

Sound waves are longitudinal waves. Sound waves travel fastest in solids, slower in liquids and slowest in gases. This means the air particles (or particles of the medium) move back and forth on paths that are parallel to the direction of wave propagation and thus take the form of compressions and rarefactions of the molecules in the air itself.



Fig. 4.4: Sound waves

Water waves

Water waves are a combination of both transverse and longitudinal waves. These waves are periodic disturbances that move away from the source and carry energy as they go.



Fig. 4.5: Water waves

Ocean waves

These waves are longitudinal waves that are observed moving through the bulk of liquids, such as our oceans. Ocean waves are powerful forces that erode and shape of the world's coastlines. Most of them are created by the wind. Winds that blow over the top of the ocean, create friction between the air and water molecules, resulting in a frictional drag as waves on the surface of the ocean.



Fig. 4.6: Ocean waves

Earthquake waves

Earthquakes occur when elastic energy is accumulated slowly within the Earth's crust (as a result of plate motions) and then released suddenly along fractures in the crust called faults. Earthquake waves are also called seismic waves and actually travel as both transverse and longitudinal waves.

The P waves (Primary waves or compressional waves) in an earthquake are examples of longitudinal waves. The P waves travel with the fastest velocity and are the first to arrive.

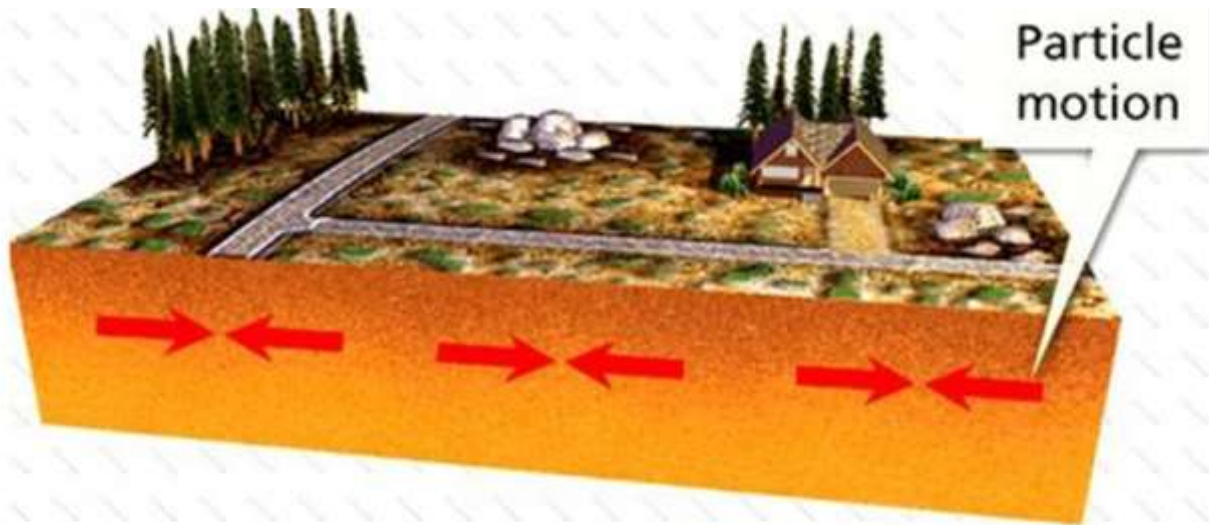


Fig. 4.7: P Seismic waves

The S waves (Secondary waves or shear waves) in an earthquake are examples of transverse waves. S waves propagate with a velocity slower than P waves, arriving several seconds later.

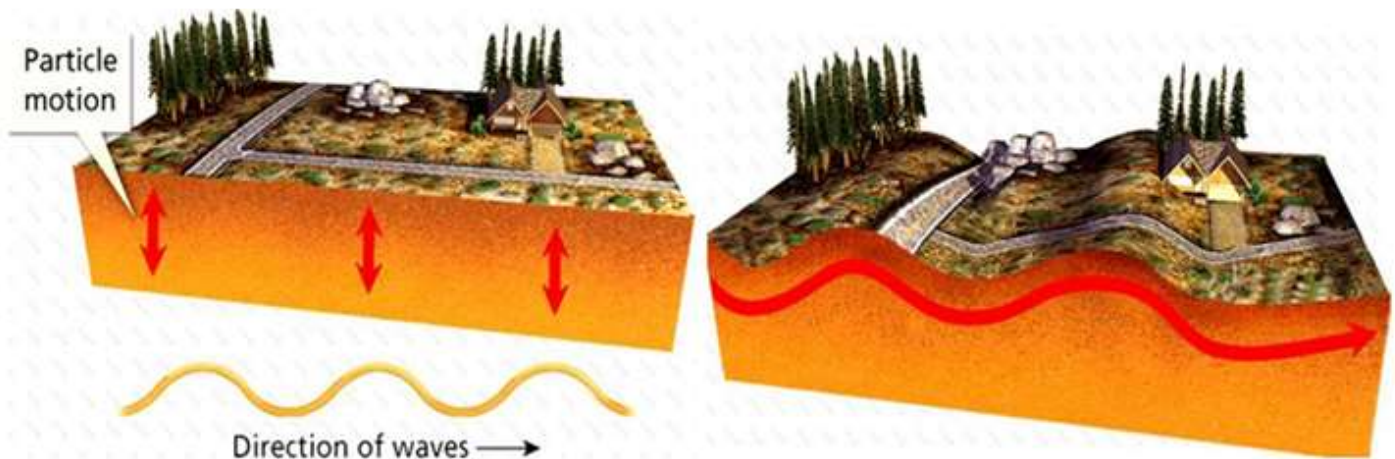


Fig. 4.8: S seismic waves

Body Waves

Body waves are of two types: compressional or primary (P) waves which are longitudinal in nature and shear or secondary (S) waves which are transverse in nature. P- and S- waves are called 'body waves' because they can travel through the interior of a body, such as the Earth's inner layers, from the focus of an earthquake to distant points on the surface. The Earth's molten core are only travelled by compressional waves.

Surface Waves

When waves occur at or near the boundary between two media, a transverse wave and a longitudinal wave can combine to form a surface wave.

Examples of surface waves are a type of seismic wave formed as a result of an earthquake and water waves.



Fig.4.9: Surface water waves.

4.3.2 Electromagnetic waves

These waves consist of disturbances in the form of varying electric and magnetic fields. No material medium is necessary for their movement and they travel more easily in vacuum than in matter.

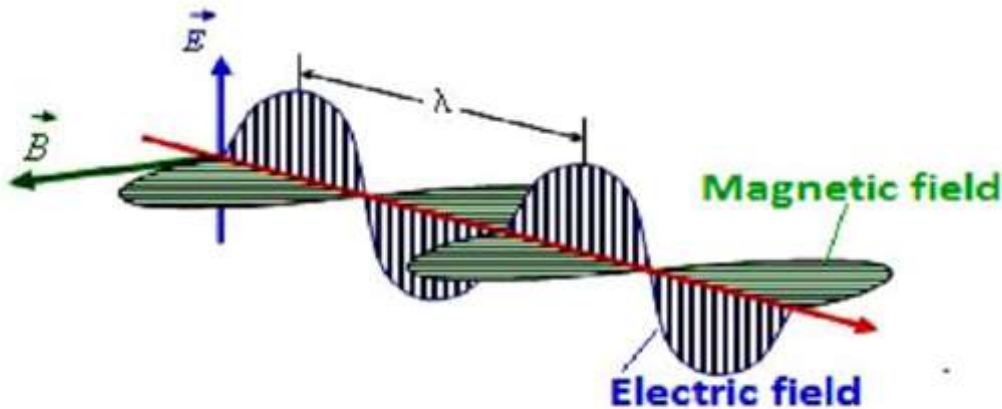


Fig. 4-10: Electromagnetic waves

Examples of electromagnetic waves are: Radio waves, Microwaves, Infrared radiation, Visible light, Ultraviolet light, X-rays and Gamma rays. These waves vary according to their wavelengths.

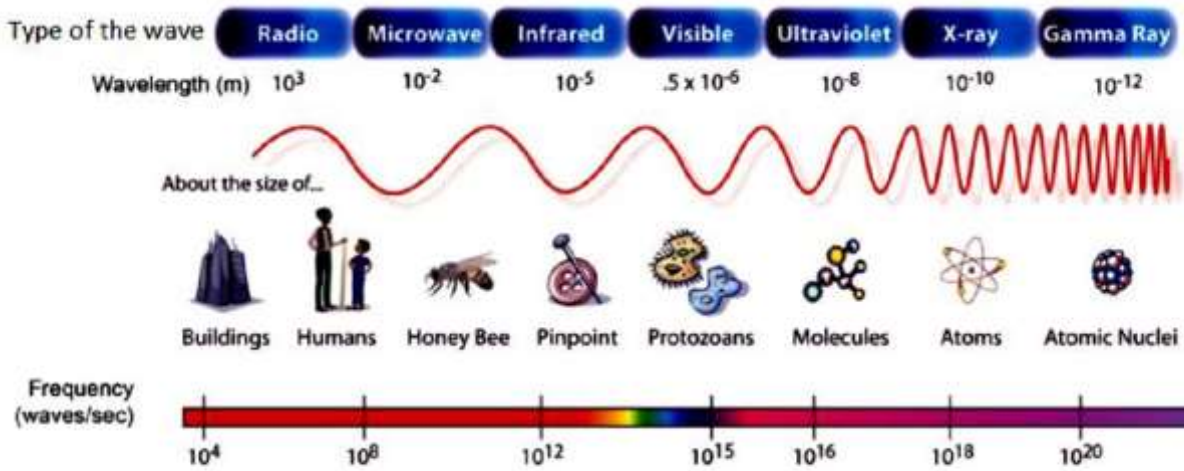


Fig. 4-11: Examples of electromagnetic waves

4.3.3 Matter Waves

If we perform the double slit diffraction experiment using a beam of electrons instead of light, we still get a diffraction pattern. The interpretation of this is that **matter travels as a wave**. Thus "matter acts as both a particle and as a wave." If we can sometimes consider an electron to be a wave, what is its wavelength? Louis de Broglie postulated that all particles with momentum have a wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

..... Equation 4-5

Where

- $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$ is Planck's constant (energy of a photon divided by its frequency),
- p is the magnitude of the momentum of the particle.

The matter waves describe the wavelike characteristics of atomic-level particles.

For mechanical waves, the speed of the wave is a property of the medium, speed does not depend on the size or shape of the wave.

Example 4.1

1. Find de Broglie wavelength for

- a) a particle of mass $m = 10^{-6} \text{ g}$ moving with a speed of $v = 10^{-6} \text{ m/s}$.
- b) a baseball of mass 0.17 kg moving at 100 km/h

answer

$$\text{a) } \lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{10^{-9} \times 10^{-6}} = 6.63 \times 10^{-19} \text{ m}$$

$$\text{b) } \lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \times 3600}{0.17 \times 100 \times 10^3} = 1.4 \times 10^{-34} \text{ m}$$

4.4 PROGRESSIVE WAVES

A progressive wave is also called a travelling wave which consists of a disturbance moving from one point to another. As a result, energy is transferred between points. Progressive mechanical waves can be categorised according to the direction of the effect of the disturbance relative to the direction of travel.

Equation of a progressive wave

An equation can be performed to represent displacement of a vibrating y particle in a medium in which a wave passes. Suppose a wave moves from left to right and that a particle at the origin moves with displacement given by equation.

$$y_A = A \sin \omega t \quad \text{..... Equation 4-6}$$

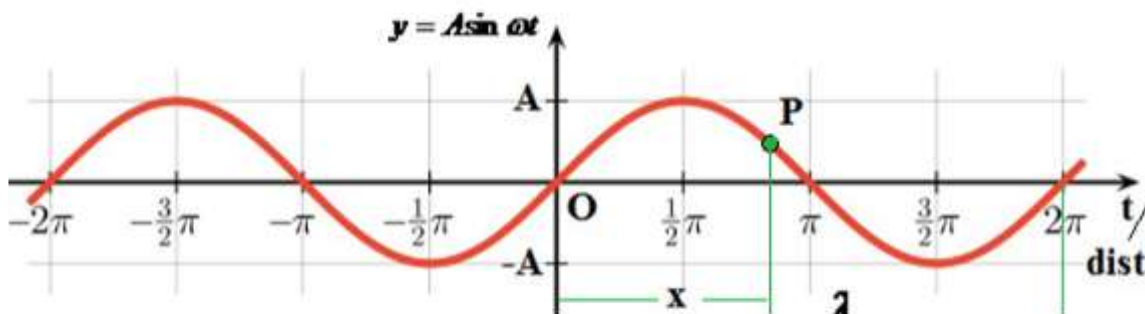


Fig. 4-12: Curve of Transverse wave

A particle at P will be out of phase from the particle at O, so, its displacement is given by;

$$y_p = A \sin(\omega t - \Phi)$$

But the phase angle

$$\Phi = \frac{2\pi x}{\lambda}$$

$$y_p = A \sin 2\pi\left(\omega t - \frac{2\pi x}{\lambda}\right) \dots\dots\dots \text{Equation 4-7}$$

Various forms of progressive wave function are listed below;

$$y_p = A \sin 2\pi\left(ft - \frac{x}{\lambda}\right) \dots\dots\dots \text{Equation 4-8}$$

$$y_p = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \dots\dots\dots \text{Equation 4-9}$$

$$y_p = A \sin \frac{2\pi}{\lambda}(vt - x) \dots\dots\dots \text{Equation 4-10}$$

$$y_p = A \sin \omega\left(t - \frac{x}{v}\right) \dots\dots\dots \text{Equation 4-11}$$

where y = displacement

A = amplitude

ω = angular frequency

f = frequency

k = propagation constant

T = time period

λ = wave length

v = wave velocity

t = instantaneous time

x = position of particle from origin

Equations 4.7 to 4.11 represent plane progressive waves. The negative sign indicates that the wave is moving from left to right and all points on the right of O will lag behind O .

A wave travelling in opposite direction; from right to left arrives at P before O . So, vibrations at P will read that at O and then displacement is given by;

$$y_p = A \sin 2\pi\left(\omega t + \frac{2\pi x}{\lambda}\right) \dots\dots\dots \text{Equation 4-12}$$

$$y_p = A \sin 2\pi \left(ft + \frac{x}{\lambda} \right) \dots\dots\dots \text{Equation 4-13}$$

$$y_p = A \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda} \right) \dots\dots\dots \text{Equation 4-14}$$

$$y_p = A \sin \omega \left(t + \frac{2\pi x}{\lambda} \right) \dots\dots\dots \text{Equation 4-15}$$

EXAMPLE 1

A travelling wave is described by the equation $y(x, t) = 0.003 \cos (20x + 200t)$ where y and x are measured in metres and t in seconds. What is the direction in which the wave is travelling? Calculate the following physical quantities:

- (a) angular wave number
- (b) wavelength
- (c) angular frequency
- (d) frequency
- (e) time period
- (f) wave speed
- (g) amplitude
- ♥ particle velocity when $x = 0.3$ m and $t = 0.02$ s
- (i) particle acceleration when $x = 0.3$ m and $t = 0.02$ s

Solution:

The equation $y(x, t) = 0.003 \cos (20x + 200t)$ has the form $y = A \sin (\omega t + \Phi)$. So, it is travelling towards- x direction.

- (a) Angular wave number or propagation constant, $k = \frac{\Phi}{x} = \frac{20x}{x} = 20 \text{ m}^{-1}$
- (b) Wavelength: $k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{20} = 0.31 \text{ m}$.
- (c) Angular frequency, $\omega = 200 \text{ rad/s}$
- (d) Frequency, $f = \frac{\omega}{2\pi} = \frac{200}{2\pi} = 32 \text{ Hz}$
- (e) Time Period, $T = \frac{1}{f} = \frac{1}{32} = 0.031 \text{ s}$
- (f) Wave speed, $v = \lambda f = 0.31 \times 32 = 10 \text{ m/s (approx.)}$
- (g) Amplitude, $A = 0.003 \text{ m}$
- (h) Given; $x = 0.3 \text{ m}$, $t = 0.02 \text{ s}$

$$\therefore v = \frac{dy}{dt} = -0.003 \times 200 \sin (20x + 200t) = -0.6 \sin 10 = +0.33 \text{ m/s}$$

- (i) Given; $x = 0.3 \text{ m}$, $t = 0.02 \text{ s}$

$$\therefore a = \frac{dv}{dt} = -0.6 \times 200 \cos (20x + 200t) = -120 \cos 10 = +101 \text{ m/s}^2$$

4.5 PRINCIPLE OF SUPERPOSITION

The displacement at any time due to any number of waves meeting simultaneously at a point in a medium is the vector sum of the individual displacements of each one of the waves at that point at the same time.

This means that when two waves travel in a medium, their combined effect at any point can be determined using this principle. Consider two waves of displacements y_1 and y_2

passing through the same medium. The resultant displacement after superposition is:

$$y = y_1 + y_2 \quad \text{..... Equation 4-17}$$

When two pulses of equal or different amplitudes on a string approach each other, then on meeting, they superimpose to produce a resultant pulse of amplitude greater than any of the two. After crossing, the two pulses travel independently.

4.5.1 Stationary waves

A stationary wave (or a standing wave) is a wave which results when two waves travelling in opposite directions and having the same speed, frequency and approximately equal amplitudes are superposed. A standing wave is shown in Fig. 4.6 below.

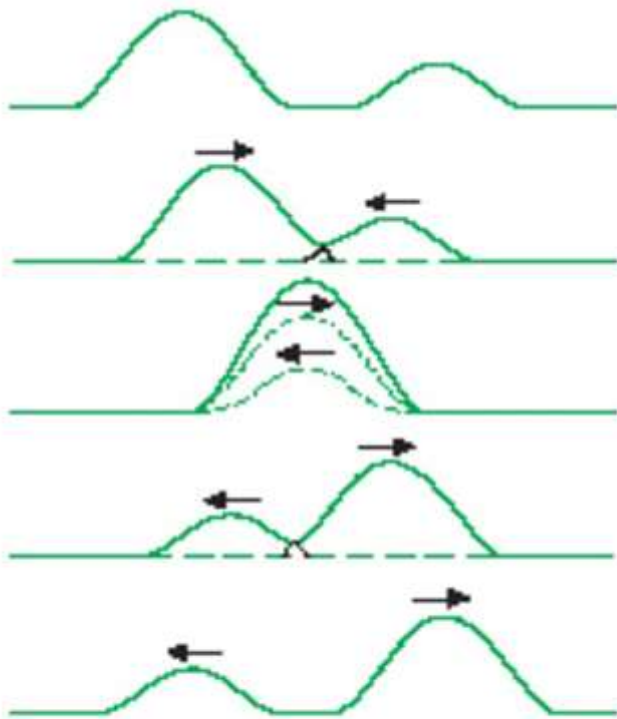


Fig. 4-14: Superposition of waves

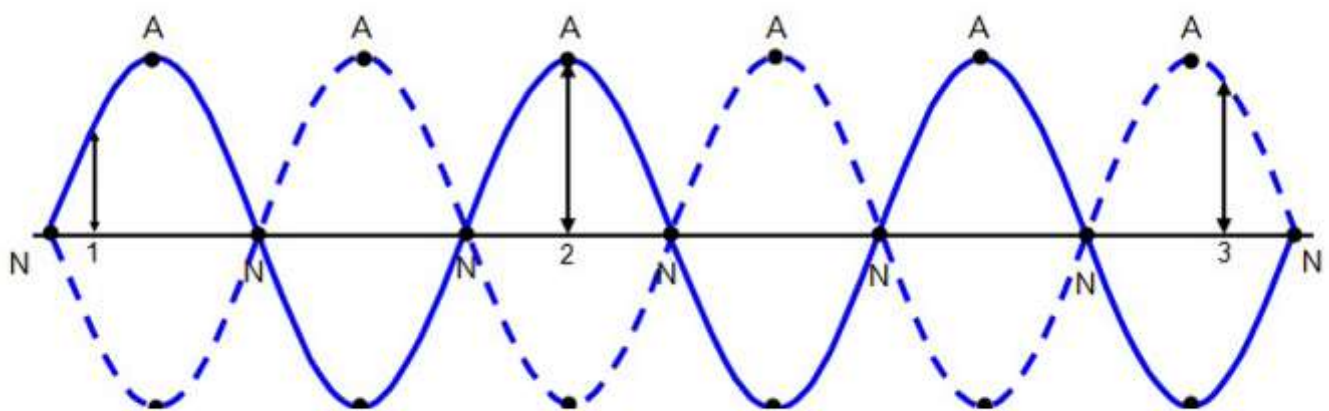


Fig. 4-15: Nodes and antinodes of a standing wave.

Here, A denotes the antinodes and N denotes the nodes.

Consider the displacement $y_1 = a \sin(\omega t - \Phi)$ of a progressive sinusoidal wave at time t and at a distance x from the origin and moving to right.

Consider also the displacement y_2 of an identical wave travelling in opposite direction given by;

$$y_2 = a \sin(\omega t + \Phi).$$

If these waves are superposed, the resultant displacement y is given by;

$$y = y_1 + y_2 \Rightarrow y = a \sin(\omega t - \Phi) + a \sin(\omega t + \Phi)$$

$\Rightarrow$

$$y = 2a \cos \Phi \sin \omega t$$

$$= 2a \cos kx \sin \omega t \quad \text{..... Equation 4-18}$$

$$[\Phi = kx, \text{Eq. (4.4)}]$$

The only variable part of equation 4.18 is $\sin \omega t$. This means that the amplitude of the resultant displacement is given by equation

$$A = 2a \cos kx \quad \text{..... Equation 4-19}$$

4.5.2 Mathematical treatment of superposition

Position of nodes

A node is defined as the point of zero amplitude. This means

$$A = 2a \cos kx = 0$$

$\Rightarrow$

$$\cos kx = 0 \quad \text{..... Equation 4-20}$$

$\Rightarrow$

$$kx = \cos^{-1}(0)$$

$\therefore$

$$kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \dots$$

$$\text{But } k = \frac{2\pi}{\lambda} \text{ [Eq. (4.4)]}$$

$\therefore$

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \frac{7\lambda}{4}, \dots$$

That is,

$$x = \frac{m\lambda}{4} \quad \text{..... Equation 4-21}$$

where $m = 1, 3, 5, 7, 9, \dots$

Equation (4.21) means that nodes are obtained when the horizontal displacement of waves are odd quarter values of wavelength.

Position of antinodes

Antinodes are points of maximum displacements. So, antinodes are obtained when the value of Equation 4.19 is maximum. This occurs when;

$$\cos kx = 1 \quad \text{..... Equation 4-22}$$

$$\Rightarrow kx = \cos^{-1}(1)$$

$$\Rightarrow kx = 0, \pi, 2\pi, 3\pi, 4\pi, 6\pi, \dots$$

$$x = 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, \frac{4\lambda}{2}, \frac{5\lambda}{2}, \frac{6\lambda}{2}, \dots$$

$$\text{That is} \quad x = \frac{n\lambda}{2} \quad \text{..... Equation 4-23}$$

where $n = 0, 1, 2, 3, 4, 5, 6, \dots$

This means that antinodes are obtained when horizontal displacement x is a multiple of $\frac{\lambda}{2}$.

Note: The separation of adjacent nodes and adjacent antinodes are $\frac{\lambda}{2}$.

$$\left(\begin{array}{c} \text{Seperation of} \\ \text{adjacent nodes} \end{array} \right) = \left(\begin{array}{c} \text{Seperation of} \\ \text{adjacent antinodes} \end{array} \right) = \frac{\lambda}{2} \quad \text{..... Equation 4-24}$$

EXAMPLE 2

Find the path difference between the two waves $y_1 = \sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$ and

$$y_2 = a_2 \cos\left(\omega t - \frac{2\pi x}{\lambda} + \phi\right) = a_2 \sin\left(\omega t - \frac{2\pi x}{\lambda} + \phi + \frac{\pi}{2}\right)$$

Solution:

$$\text{Phase difference} = \Delta\phi = \left(\omega t - \frac{2\pi x}{\lambda} + \phi\right) - \left(\omega t - \frac{2\pi x}{\lambda}\right) = \phi + \frac{\pi}{2}$$

$$\text{Path difference} = \Delta x = \frac{2\pi}{\lambda} \Delta\phi = \frac{2\pi}{\lambda} \left(\phi + \frac{\pi}{2}\right)$$

4.6 PROPERTIES OF WAVES

This section introduces the properties of waves and wave motion to describe the behaviour of waves in detail.

4.6.1 Reflection

This is the property of waves to bounce back from the surface on which they hit. Huygens principle can also be applied to reflection. Consider a parallel beam of light incident on the reflecting surface such that its direction of travel makes an angle i with the normal to the surface.

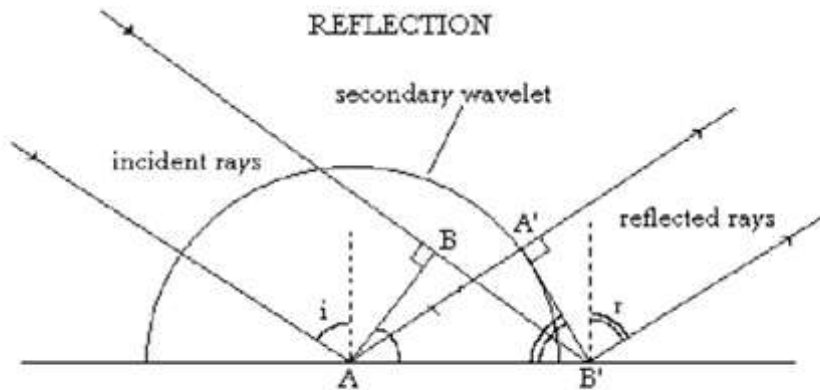


Fig. 4-15: Reflection of waves

Consider that side A of an associated wavefront AB has just reached the surface. In the time that light from side B of the wavefront travels to B', a secondary wavelet of radius equal to BB' will be generated by A. Because of the reflecting surface, this wavelet is a semicircle above the surface. The new wavefront generated by reflection will be the tangent to this wavelet and will also contain point B'. The reflected wavefront will be A'B'.

Triangles ABB' and $B'A'A$ are congruent as

$$\angle A'AB' = \angle BBA'$$

..... Equation 4-25

$$\angle A'AB' = 90 - r$$

$$\angle BBA' = 90 - i$$

$$90 - r = 90 - i$$

$\therefore$

$$i = r$$

..... Equation 4-26

We conclude by saying that all laws of reflection are obeyed. So, any wavefront can reflect.

4.6.2 Refraction

Consider a parallel beam of waves (for example light waves) incident on a refracting surface between two media such that its direction of travel

makes angle θ_1 with the normal to the refracting surface.

Consider side A of the wavefront AB has reached the surface before B . If the ray from the other side B of the beam consequently travels to C at time t ,

$$BC = C_1 t \quad \text{..... Equation 4-27}$$

Assuming: $C_1 \rightarrow$ speed of light in medium 1

$C_2 \rightarrow$ speed of light in medium 2

$t \rightarrow$ time taken for the ray to move from B to C

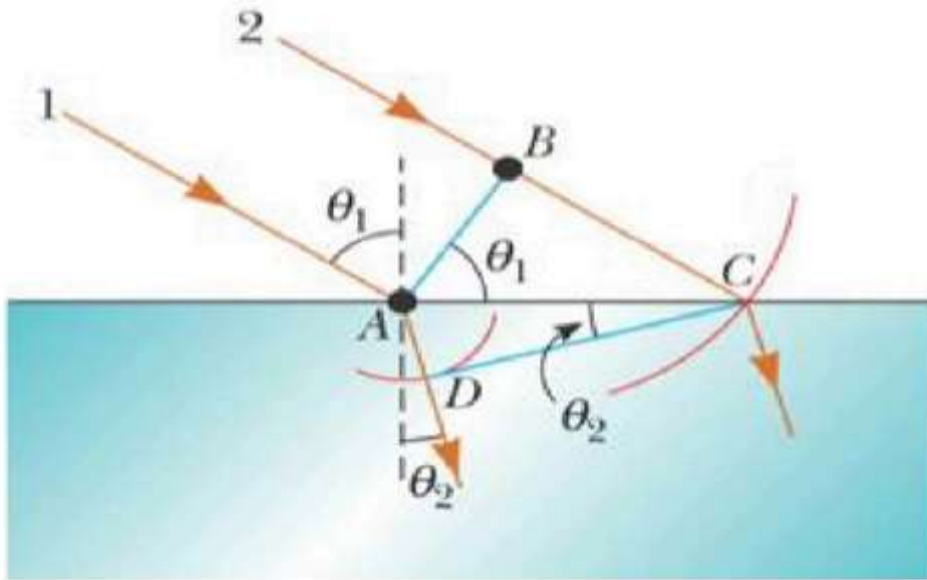


Fig. 4-16: Refraction of waves

At the same time, wavelets from A travel distance AD in medium 2. Here, a refracted wavefront CD is formed by many wavelets in the beam. Fig.4-16 above illustrates this description.

$$AD = C_2 t \quad \text{..... Equation 4-28}$$

Considering triangle ABC and ADC :

$$\sin \theta_1 = \frac{BC}{AC} = \frac{C_1 t}{AC} \quad \text{..... Equation 4-29}$$

$$\sin \theta_2 = \frac{AD}{AC} = \frac{C_2 t}{AC} \quad \text{..... Equation 4-30}$$

Dividing Equation 4.29 by Equation 4.30 gives:

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{C_1}{C_2} \quad \text{..... Equation 4-31}$$

By Snell's law:

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{C_1}{C_2} = \text{Constant} \quad \text{..... Equation 4-32}$$

Equation 4-32 confirms Snell's law meaning that waves behave like normal light during reflection.

4.6.3 Interference

In the region where wave trains from coherent sources (sources of the same frequency) cross, superposition occurs giving reinforcements of waves at some points which is called constructive interference and cancellation at others which is called destructive interference. The resulting effect is called interference pattern or the system of fringes.

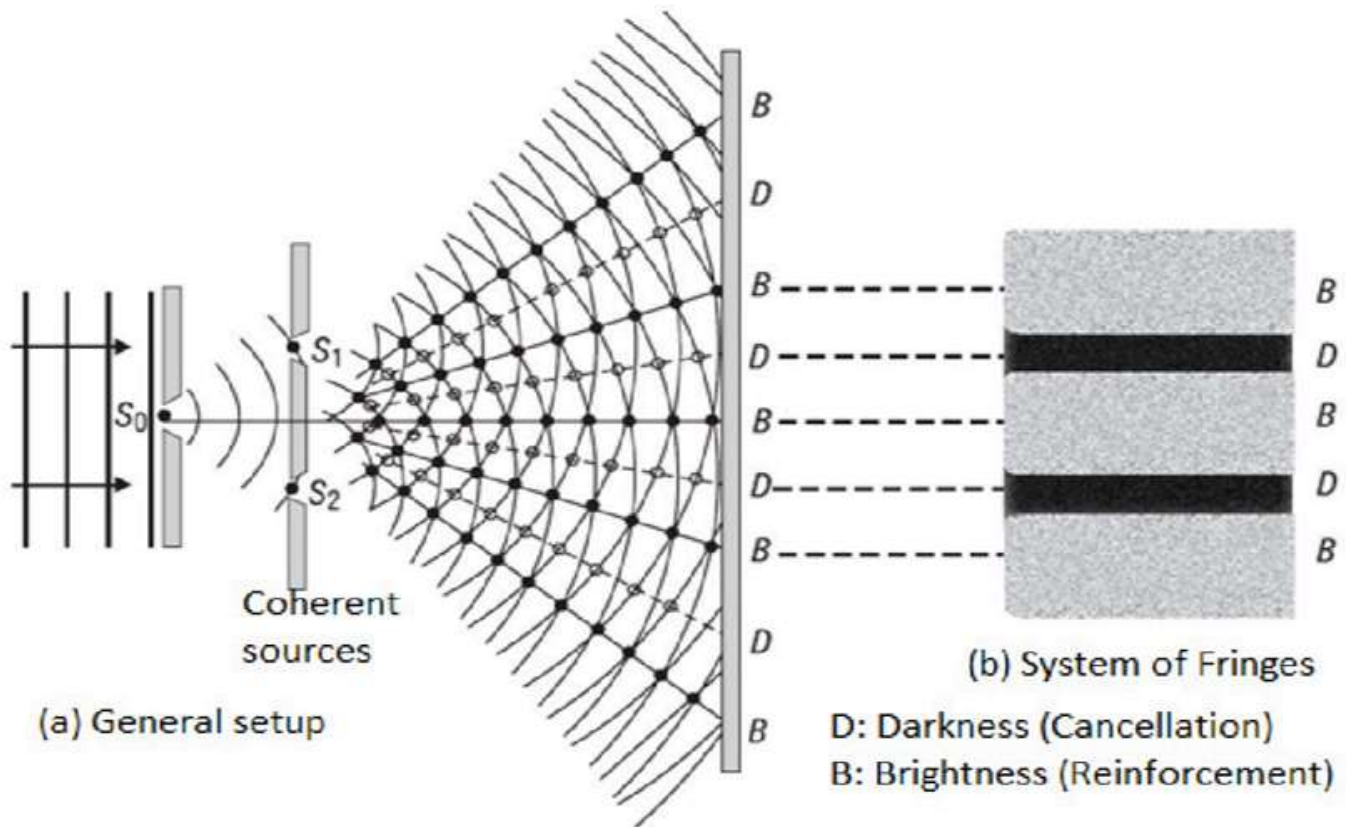


Fig.4.17; Interference of waves

4.6.4 Diffraction

This is a phenomenon in which waves from one source meet an obstacle and spread around it. Diffraction is normally observed when these waves pass through narrow slits. There are two types of diffraction and these are; Fresnel's diffraction and Fraunhofer diffraction.

a) Fresnel's diffraction

This is a type of diffraction in which either the source of waves or screen on which diffraction is observed or both are at finite distances from the obstacle that cause diffraction. Below are different cases to explain this diffraction.

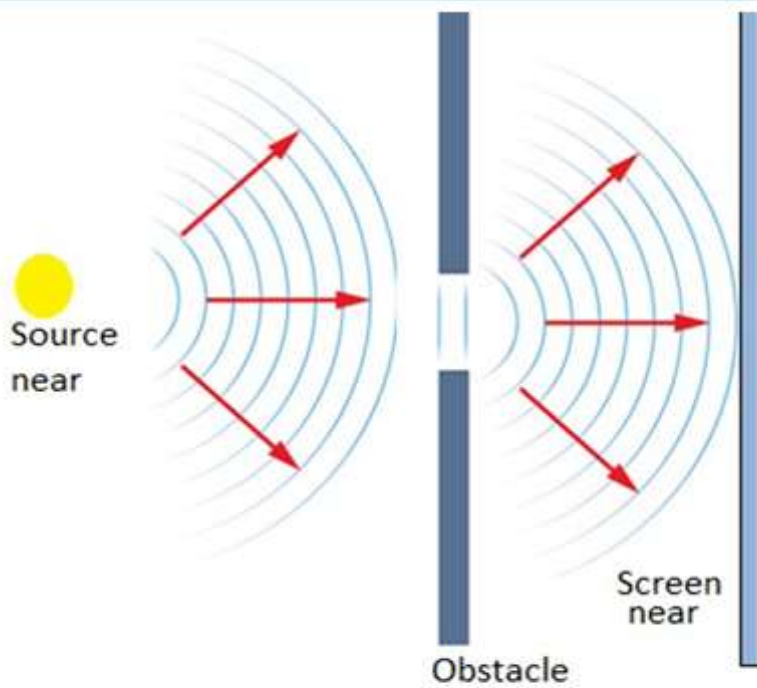


Fig. 4.18: Fresnel's diffraction case 1

Case 1: the source and the screen placed at finite distances.

Case 2: the source is placed at infinite distance from obstacle and the screen is near.

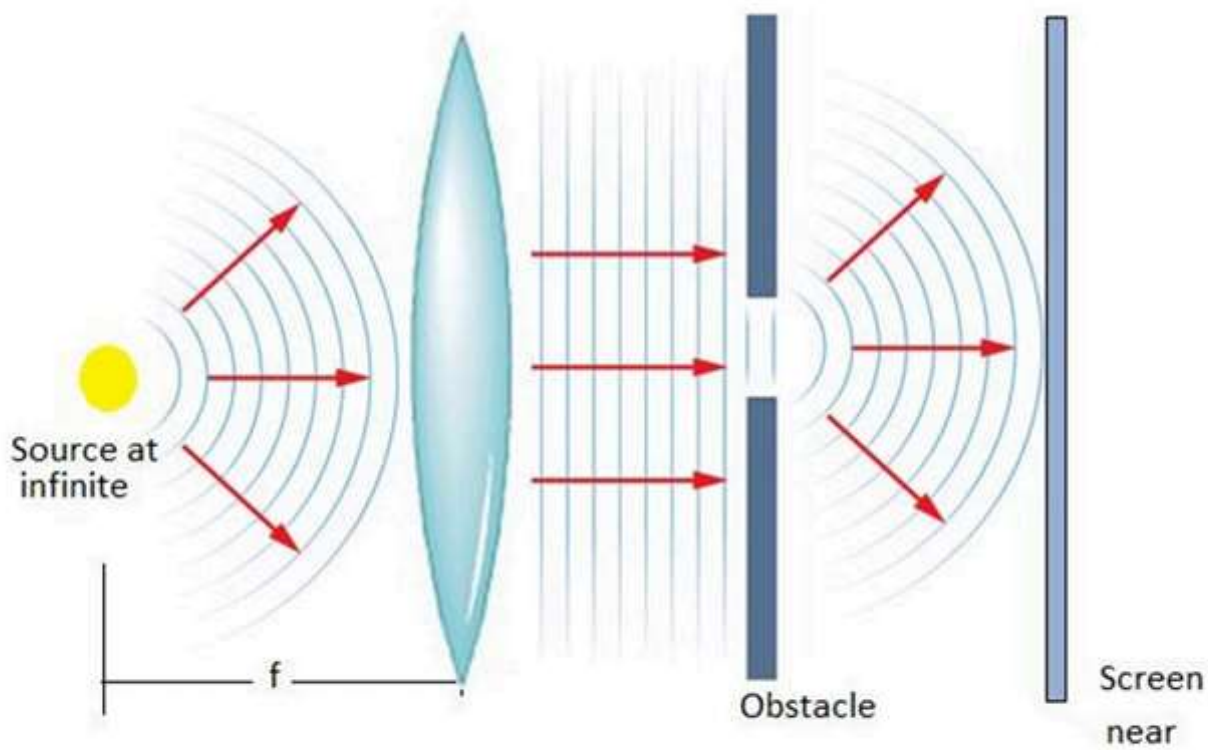


Fig. 4.19: Fresnel's diffraction case 2

Case 3: the screen is placed at infinite distance from obstacle and the source is near.

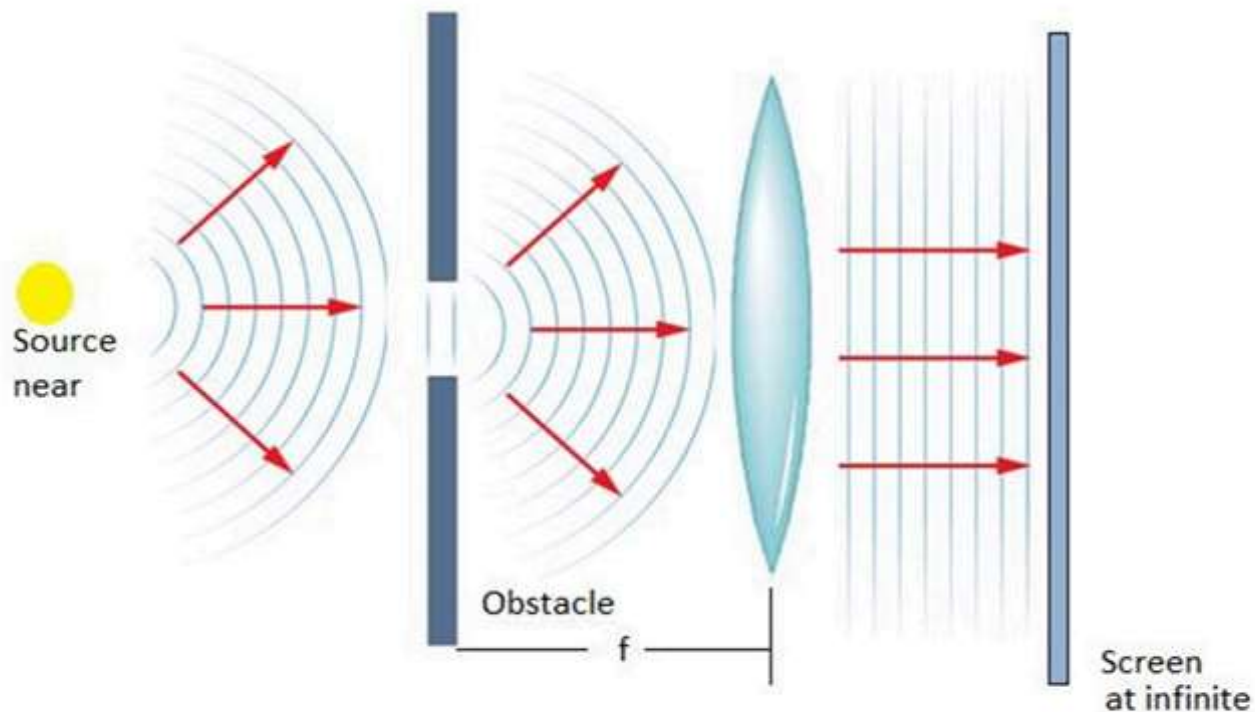


Fig. 4.20: Fresnel's diffraction case 3.

b) Fraunhofer Diffraction

This is a type of diffraction in which the source of waves and the screen on which diffraction is observed are effectively at infinite distances from the obstacle. This phenomenon is practically complicated but theoretically understood. To obtain waves to or from infinite source in laboratory, biconvex lenses are used.

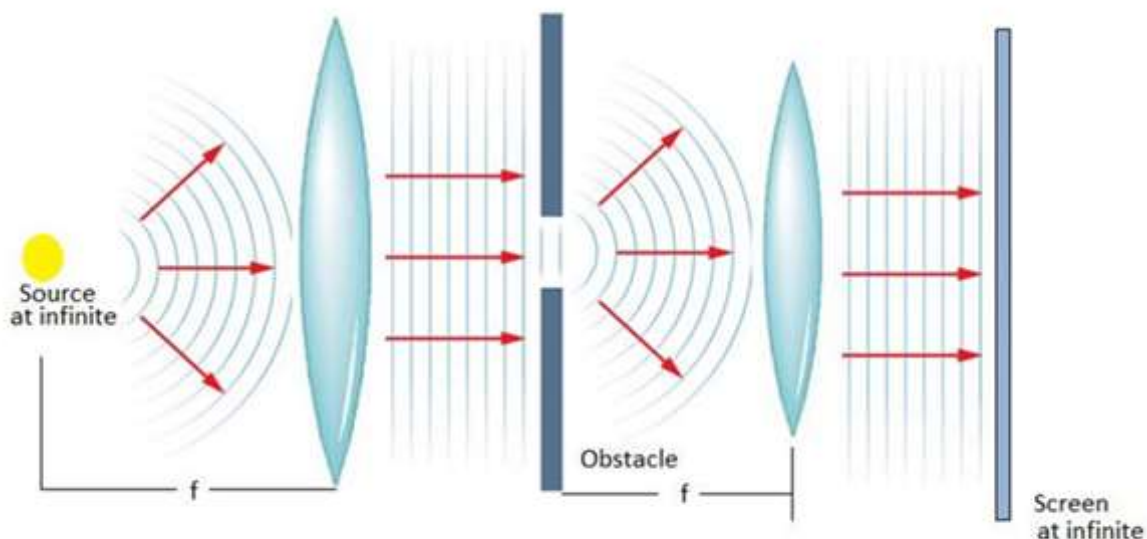


Fig. 4.21: Fraunhofer diffraction

4.7 WAVE ON A VIBRATING STRING

ACTIVITY 4-1: Propagation of Waves

Learning Objectives

- To observe the propagation of vibrations through a solid
- To understand how sound is transmitted through a medium

Required Materials

Spoon, string of length 1 m

Procedure

- (a) Tie the spoon into the middle of the length of string so that it will hang freely when you hold the string ends.
- (b) Hold the string ends to your temples or the bone just under your ears as you strike the spoon with a pen or other object.

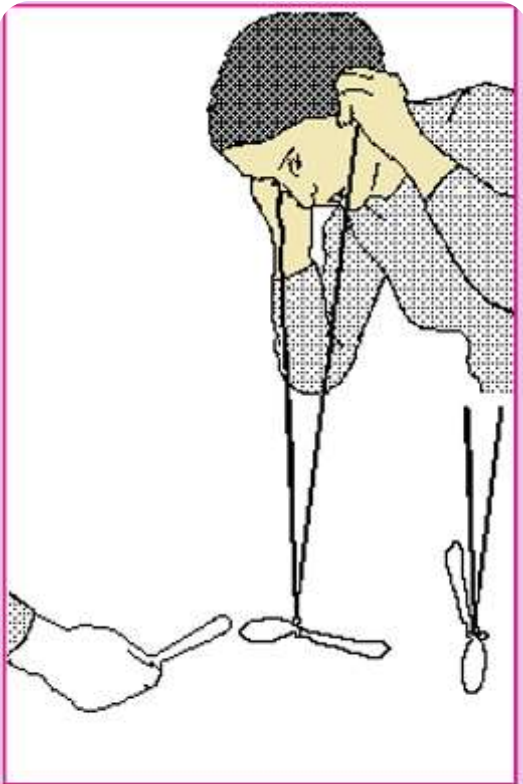


Fig. 4.22: Second overtone

Discussion Questions

1. What causes the sound to be loud when the string is held to your head?
2. Why does the bone in front of your ear transmit vibrations more easily than other bones?
3. What is the purpose of the string in this activity?

Standing wave also known as a **stationary wave**, is wave pattern that results when two waves of the same frequency; wavelength and amplitude travel in opposite directions along string and interfere.

The point at which the two waves cancel are called **node**. There no motion in the string at the nodes, but midway between two adjacent nodes, the string vibrates with the largest amplitude. These points are called **antinodes**. At points between successive nodes the vibrations are in **phase**.

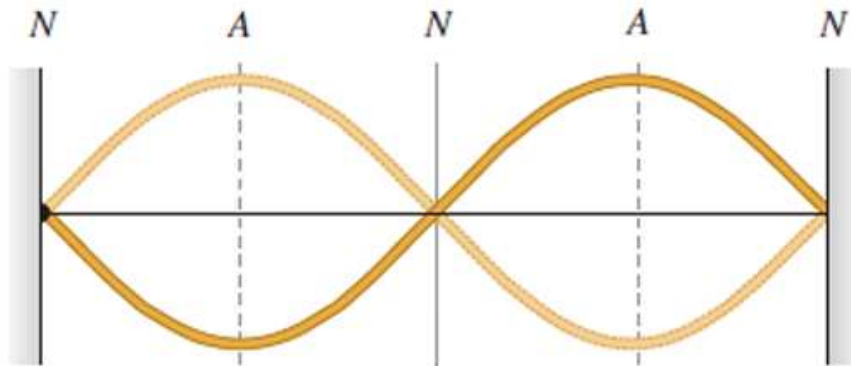


Fig.4. 23 Standing wave with 3 nodes Standing-wave produced at various times by two waves of equal amplitude traveling in opposite directions. For the resultant wave y , the nodes are points of zero displacement, and the antinodes are points of maximum displacement.

A single loop corresponds to either a crest or trough alone, while two loops correspond to a crest and trough together, or one wave length.

Stationary waves are present in the vibrating strings of musical instruments. A violin string, for instance, when bowed or plucked, vibrates as a whole, with nodes at the ends, and also vibrates in halves, with a node at the center, in thirds, with two equally spaced nodes, and in various other fractions, all simultaneously. The vibration as a whole produces the **fundamental tone**, and the other vibrations produce the **various harmonics**.

Standing waves can occur at more than one frequency. The lowest frequency of oscillation that produces a standing wave gives rise to the pattern shown in Fig. 4.24b. The standing waves shown in Figs. 4.24c and 4.24d are produced at precisely twice and three times the lowest frequency,

respectively, assuming the tension in the cord is the same. The cord can also oscillate with four loops (four antinodes) at four times the lowest frequency, and so on.

The frequencies at which standing waves are produced are the natural **frequencies** or **resonant frequencies** of the cord, and the different standing wave patterns shown in Fig. 4.24 are different "resonant modes

of vibration." A standing wave on a cord is the result of the interference of two waves traveling in opposite directions. A standing wave can also be considered a vibrating object at resonance. Standing waves represent the same phenomenon as the resonance of an oscillating spring or pendulum, However, a spring or pendulum has only one resonant frequency, whereas the cord has an infinite number of resonant frequencies, each of which is a whole-number multiple of the lowest resonant frequency.

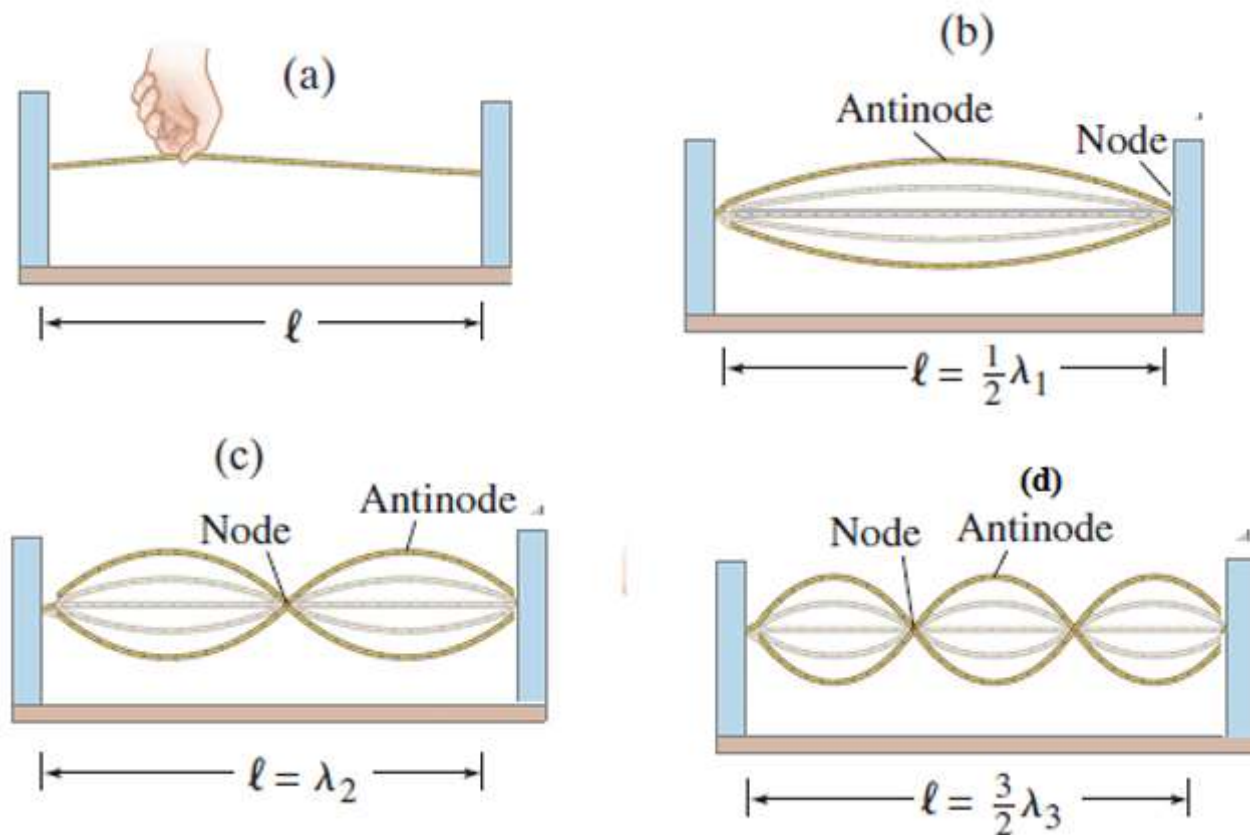


Fig.4. 24(a) A string is plucked; (b) Fundamental or first harmonic

$f_0 = \frac{v}{\lambda}$; **(c) First overtone or second harmonic** $f_1 = 2f_0$; **(d) Second overtone or third harmonic** $f_2 = 3f_0$

To determine the **resonant frequencies**, we first note that the wavelengths of the standing waves bear a simple relationship to the length of

the string and use Eq4.06 $\lambda = \frac{v}{f}$.

The lowest frequency, called the **fundamental frequency**, corresponds to

one antinode (or loop). And as can be seen in Fig. 4.24b, the whole length corresponds to one-half wavelength.

$$\text{Thus } l = \frac{\lambda_1}{2} \Leftrightarrow \lambda_1 = 2l$$

$$\text{Fundamental frequency or 1}^{\text{st}} \text{ harmonic: } f_o = \frac{v}{\lambda_1} = \frac{v}{2l},$$

where λ_1 stands for the wavelength of the fundamental frequency.

The other natural frequencies are called **overtones**; for a vibrating string they are whole-number (integral) multiples of the fundamental, and then are also called **harmonics**, with the fundamental being referred to as the first harmonic. The next mode of vibration after the fundamental has two loops and is called the **second harmonic** (or first overtone), Fig. 4.24c. The length of the string at the second harmonic corresponds to one complete

wavelength: $l = \lambda_2$

1st overtone or 2nd harmonic $f_1 = \frac{v}{\lambda_2} = \frac{v}{l} = 2f_0$

2nd overtone or 3rd harmonic $f_2 = \frac{v}{\lambda_2} = \frac{3v}{2l} = 3f_0$

For the 3rd and 4th harmonics $l = \frac{3\lambda_3}{2} \Leftrightarrow \lambda_3 = \frac{2l}{3}$, and $l = \frac{4\lambda_4}{2} \Leftrightarrow \lambda_4 = \frac{l}{2}$ respectively, and so on.

3rd overtone or 4th harmonic $f_3 = \frac{v}{\lambda_3} = \frac{4v}{2l} = 4f_0$ and $f_4 = \frac{v}{\lambda_4} = \frac{5v}{2l} = 5f_0$

In general, we can write $l = \frac{n\lambda_n}{2} \Leftrightarrow \lambda_n = \frac{2l}{n}$

n^{th} Overtone or $(n+1)^{\text{th}}$ harmonic $f_n = \frac{v}{\lambda_n} = \frac{(n+1)v}{2l} = (n+1)f_0$

To find the frequency f of each vibration and see that

The wave velocity in the string with its mass per unit length $\mu = \frac{m}{l}$ and

under tension T is given by $v = \sqrt{\frac{T}{\mu}}$

A **normal mode** of an oscillating system is a motion in which all particles of the system move sinusoidally with the same frequency

EXAMPLE 4

A wire of length 400 mm and mass $1.2 \times 10^{-3} \text{ kg}$ is under a tension of 120 N. What is:

- (a) the fundamental frequency of vibration?
- (b) the frequency of the third harmonic?

Answer

$$\text{Fundamental frequency: } f_0 = \frac{1}{2 \times 400 \times 10^{-3}} \sqrt{\frac{120 \times 400 \times 10^{-3}}{1.2 \times 10^{-3}}} = 250 \text{ Hz}$$

$$\text{Third harmonic } f_2 = 3f_0 = 3 \times 250 \text{ Hz} = 750 \text{ Hz}$$

EXAMPLE 4

A wire of length 400 mm and mass $1.2 \times 10^{-3} \text{ kg}$ is under a tension of 120 N. What is

- (a) the fundamental frequency of vibration?
- (b) the frequency of the third harmonic?

Solution:

- (a) The fundamental frequency is obtained by substituting $n = 1$ in the equation 4.42.

$$f_1 = \frac{1}{2 \times 400 \times 10^{-3}} \sqrt{\frac{120 \times 400 \times 10^{-3}}{1.2 \times 10^{-3}}} = 250 \text{ Hz}$$

- (b) For third harmonic, using equation (4.37)

$$f_3 = 3f_1 = 3 \times 250 = 750 \text{ Hz}$$

Application Activity 4.2

Use the following descriptions in waves and fill the puzzle

Down:

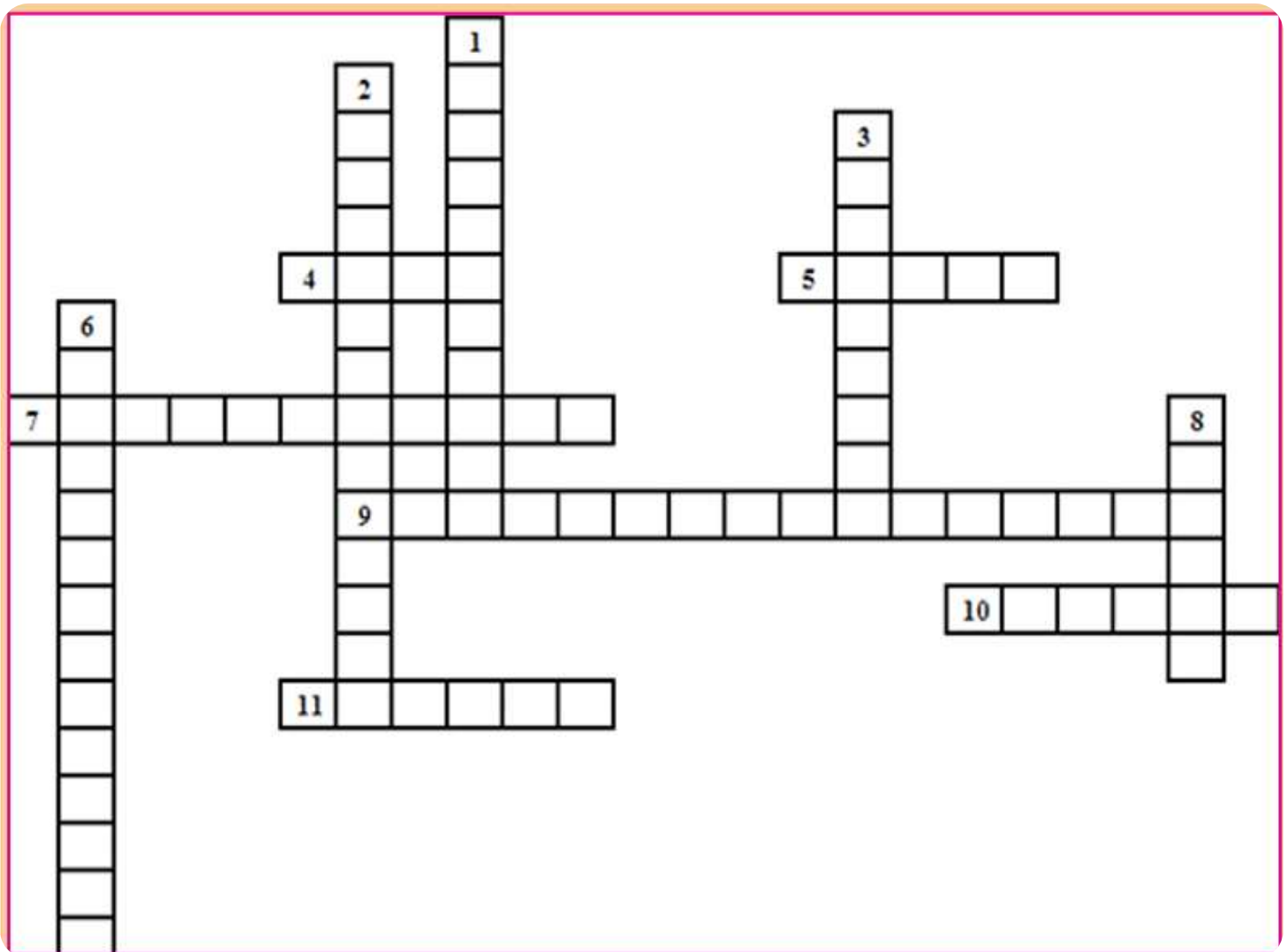
- 1) The part of a longitudinal wave where the particles of the medium are close together.
- 2) A wave which needs to travel through a medium.
- 3) A repeated back-and-forth or up-and-down motion.
- 6) A wave which moves the medium in a direction across the direction.

the energy is traveling.

8) The ability to do work.

Across:

- 4) A disturbance that transfers energy from place to place.
- 5) The highest point of a wave.
- 7) The part of a longitudinal wave where the particles of the medium are far apart.
- 9) A wave which moves the medium in the same direction as the energy is traveling.
- 10) The lowest part of a transverse wave.
- 11) The material through which a wave travels.



END OF UNIT PROJECT

Materials to choose from:

3 white screens, 3 biconvex lenses 🟡 , 3 biconcave lenses 🟡 , 3 biconvex mirrors 🟡 , 3 biconcave mirrors 🟡 , 3 boards with a hole, 3 laser pens, 3 big

torches, 3 very bright open lamps, 1 plane mirror.

The question:

Explain how you can perform Fresnel's diffraction and Fraunhofer diffraction in the laboratory.

Hypothesis:

Write a hypothesis about how diffraction is obtained in the lab.

Procedure

1. Decide which materials you will need (from the list) to test the hypothesis.
2. Plan your investigation.
 - a. Which arrangements best gives the idea of diffraction?
 - b. Which adjustments do you care to take care of ?
3. Write a procedure and show it to your teacher. Do not proceed any further until it is approved.
4. Carry out your investigation.

Collecting Data

Make sure you have recorded at least the following information:

- ◇ the hypothesis
- ◇ your procedure

Analyzing and Interpreting

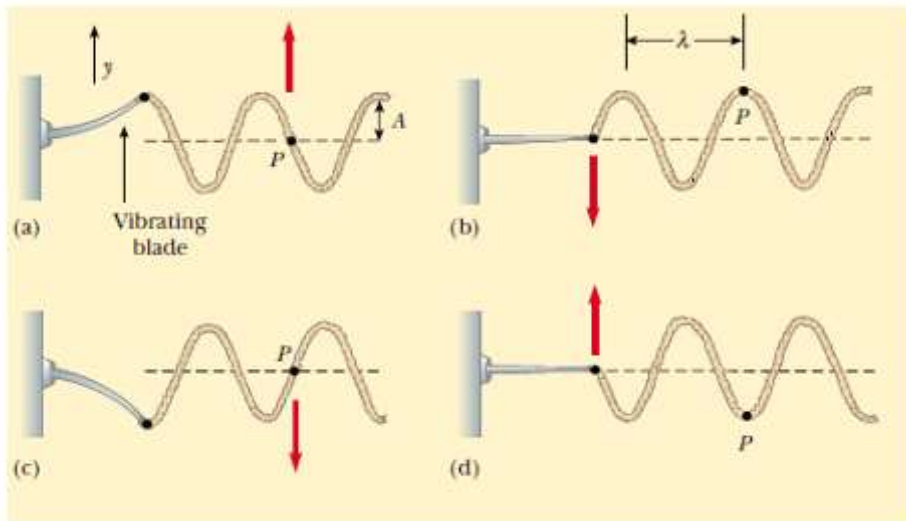
Share and compare your results with your classmates. Which idea is important to be used and achieve the proper arrangement of apparatus to achieve your objective?

Forming Conclusions

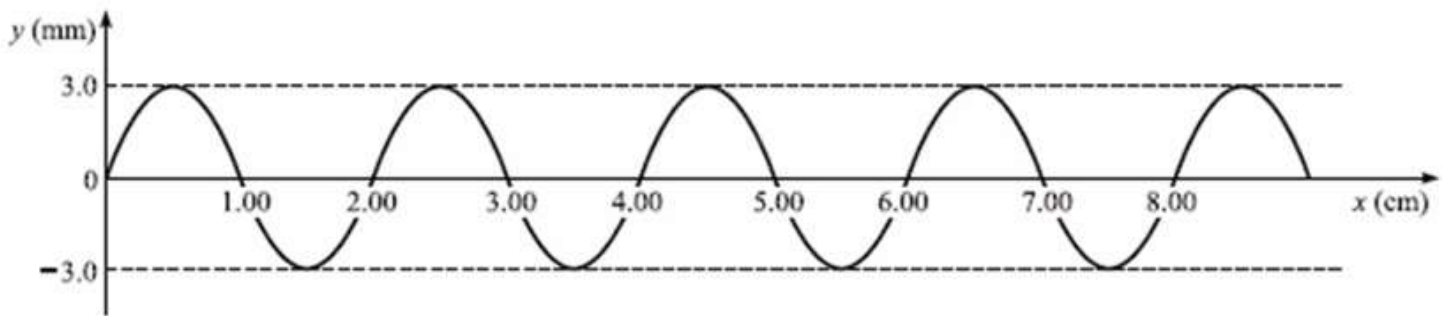
Make a brief report of your project with neat diagrams. In this project what is needed is the concept not the analysis of the fringes formed.

END OF UNIT ASSESSMENT

1. The string shown in Figure below is driven at a frequency of 5.00 Hz. The amplitude of the motion is 12.0 cm, and the wave speed is 20.0 m/s. Determine the angular frequency and wave number k for this wave, and write an expression for the wave function.



2. The wave shown in Fig. below is being sent out by a 60 Hz vibrator.

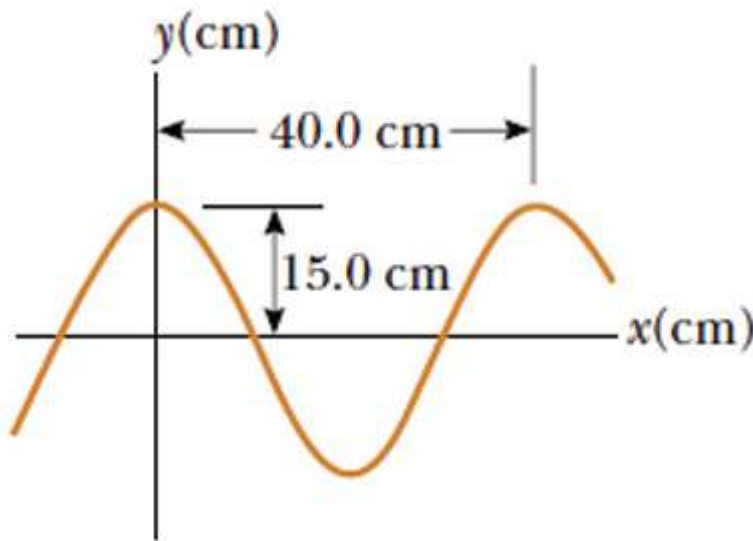


3. A string of length 3 m and mass density 0.0025 kg/m is fixed at both ends. One of its resonance frequencies is 252 Hz. The next higher resonance frequency is 336 Hz. Find the fundamental frequency and the tension in the string.

4. A wire of length 400 mm and mass 1.2×10^{-3} kg is under a tension of 120 N. What is

- the fundamental frequency of vibration?
- the frequency of the third harmonic?

5. A sinusoidal wave traveling in the positive x direction has an amplitude of 15.0 cm, a wavelength of 40.0 cm, and a frequency of 8.00 Hz. The vertical position of an element of the medium at $t = 0$ and $x = 0$ is also 15.0 cm, as shown in Figure below.



- (A) Find the wave number k , period T , angular frequency ω and speed v of the wave.
- (B) Determine the phase constant ϕ and write a general expression for the wave function.

UNIT SUMMARY

Waves can be defined as a disturbance in a material medium that transfers energy from one place to another.

The time **period (T)** of the wave is the time it takes for one complete vibration of the wave.

The **frequency f** is the number of wavelengths that pass a point in space in one second.

The **wavelength λ** is the horizontal distance in space between two nearest points that are oscillating in phase.

The **wave speed v** is the speed at which the wave advances.

Phase difference (phase angle) is the angular difference between two points on the wave or between two waves.

The **wave number** also called the propagation number k is the spatial frequency of a wave.

The **Intensity** of a wave or the power radiated by a source are proportional to the square of the amplitude.

Wavefront is a line or surface in the path of the wave motion on which the disturbance at every point have the same phase.

Mechanical waves are waves produced by the disturbance in a material medium.

A **progressive wave** consists of a disturbance moving from one point to another.

Longitudinal wave propagates through some medium with vibrations in the direction of propagation of the disturbance.

In **Transverse waves**, the direction of vibrations is perpendicular to the direction of propagation of the wave.

Equation of a progressive wave is given by:

$$y_p = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right)$$

Principle of superposition states that the resultant displacement at any time is the vector sum of the individual displacements.

Stationary waves are waves which seem to be at rest.

The positions of nodes are $x = \frac{m\lambda}{4}$ where $m = 1, 3, 5, 7, 9, \dots$

The positions of antinodes are $x = \frac{n\lambda}{2}$ where $n = 0, 1, 2, 3, 4, 5, 6, \dots$

Electromagnetic waves are disturbances in form of varying electric and magnetic fields.

All kinds of waves reflect, refract, interfere and also spread around the obstacle.

Other than the superposition of waves meeting at a point, other **conditions**

for interference are:

- The sources of the waves must be coherent, which means they emit identical waves with a constant phase difference.
- The waves should be monochromatic - they should be of a single wavelength.