

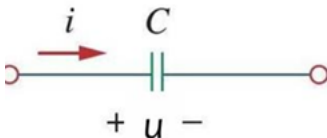
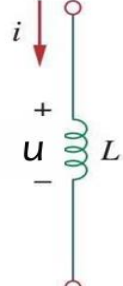
SOLUTION OF QUESTIONS CAT FOR WAVES AND OPTICS

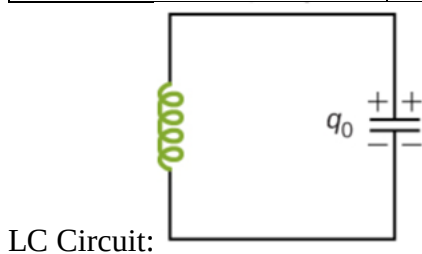
SECTION A:

- Faraday's law (Maxwell Equation of induction which states that $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$, where $\mathbf{B}$ is the Magnetic field and $\mathbf{E}$ is electric field and t is the time means that the magnetic field induces electric field in opposite direction.

This is the principle behind electrical generator in electrical energy generation and the transformer in electrical energy transmission and distribution.

2.

#	Capacitance	Inductance
Symbol		
Unity	Farads (F)	Henrys (H)



SECTION B:

1.

Restoring force: A spring or other force pulls an object back. We write this as $F = -kx$, where k is a positive constant and x is the distance from the center.

Newton's law: Force equals mass times acceleration $F = ma$. Acceleration is the second derivative of position

$$a = \frac{d^2x}{dt^2} \text{ . so } F = m \frac{d^2x}{dt^2}$$

Equalizing two forces: $m \frac{d^2x}{dt^2} = -kx$

Move everything to one side to get $m \frac{d^2x}{dt^2} + kx = 0$

Dividing with m we get: $\frac{d^2x}{dt^2} + \frac{k}{m}x = 0, \frac{k}{m} = \omega^2$

Then we get $\frac{d^2x}{dt^2} + \omega^2x = 0$

2.

The equivalence of the two systems is clear. To go from the mechanical to the electromagnetic system, we simply replace m by L , v by i , k by $1/C$, and x by q . Now $x(t)$ is given by

$$x(t) = A \cos(\omega t + \theta)$$

where $\omega = \sqrt{k/m}$ equivalent to $\omega = \sqrt{1/LC}$. Hence, the charge on the capacitor in an LC circuit is given by

$$q(t) = q_0 \cos(\omega t + \theta)$$

3.

The velocity (v):

$$v = \frac{dx}{dt} = A\omega \sin(\omega t + \theta)$$

$$i(t) = \frac{dq(t)}{dt} = -\omega q_0 \sin(\omega t + \theta)$$

SECTION C

1.

Solution:

$$\omega = \sqrt{\frac{1}{LC}}$$

$$\omega = \sqrt{\frac{1}{(2 \times 10^{-2}H)(8 \times 10^{-6}F)}} = 2.5 \times 10^3 \text{ rad/s}$$

The current (I_0) is at its maximum when all the energy is stored in the inductor. From the law of energy conservation:

$$\frac{1}{2} \frac{q_0^2}{C} = \frac{1}{2} L I_0^2$$

$$I_0 = \sqrt{\frac{1}{LC}} q_0 = \omega q_0$$

$$I_0 = 3 \times 10^{-2} A$$

This result can also be found by an analogy to simple harmonic motion, where current and charge are the velocity and position of an oscillator.

3) The capacitor becomes completely discharged in one-fourth of a cycle, or during a time $T/4$, where T is the period of the oscillations. Since $T = \frac{2\pi}{\omega}$

$$T = \frac{2\pi}{2.5 \times 10^3 \text{ rad/s}} = 2.5 \times 10^{-3} \text{ s}$$

4) The time taken for the capacitor to become fully discharged is $\frac{T}{4} = 6.3 \times 10^{-4} \text{ s}$

5) The capacitor is completely charged at $t = 0$, so, $q(0) = q_0$

$q(0) = q_0 \cos \theta$ for $\theta = 0$,

$$q(t) = 1.2 \times 10^{-5} \text{ C} \cos 2.5 \times 10^3 t$$

2.

$$(a) \int_{\text{closed surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$$

$$(b) \int_{\text{closed surface}} \vec{B} \cdot d\vec{A} = 0$$

$$(c) \oint \vec{B} \cdot d\vec{\ell} = \mu_0 i + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

$$(d) \oint \vec{E} \cdot d\vec{\ell} = - \frac{d\Phi_B}{dt}$$

The Gauss Equation of magnetism, which states that the divergence of the magnetic field is equal to zero, means that the magnetic field can be created starting from anything,

There is no specific thing or quantity required to generate a magnetic field.

An example is the creation of a magnetic field by floating any two solid materials some times done by kids. Another fact is that when a magnet is divided in any number of pieces, each of the piece acts as a magnet with two poles.

