Concepts of Electricity

by: Dr. Prosper NDIZIHIWE, INES Ruhengeri

Concepts and laws

- **1.1 Introduction: Current and Voltage**

  1.2 Circuit Elements

  1.3 Power and Energy

  1.4 Laws: Ohm's Law, Kirchhoff's Laws

  1.5 Transformations

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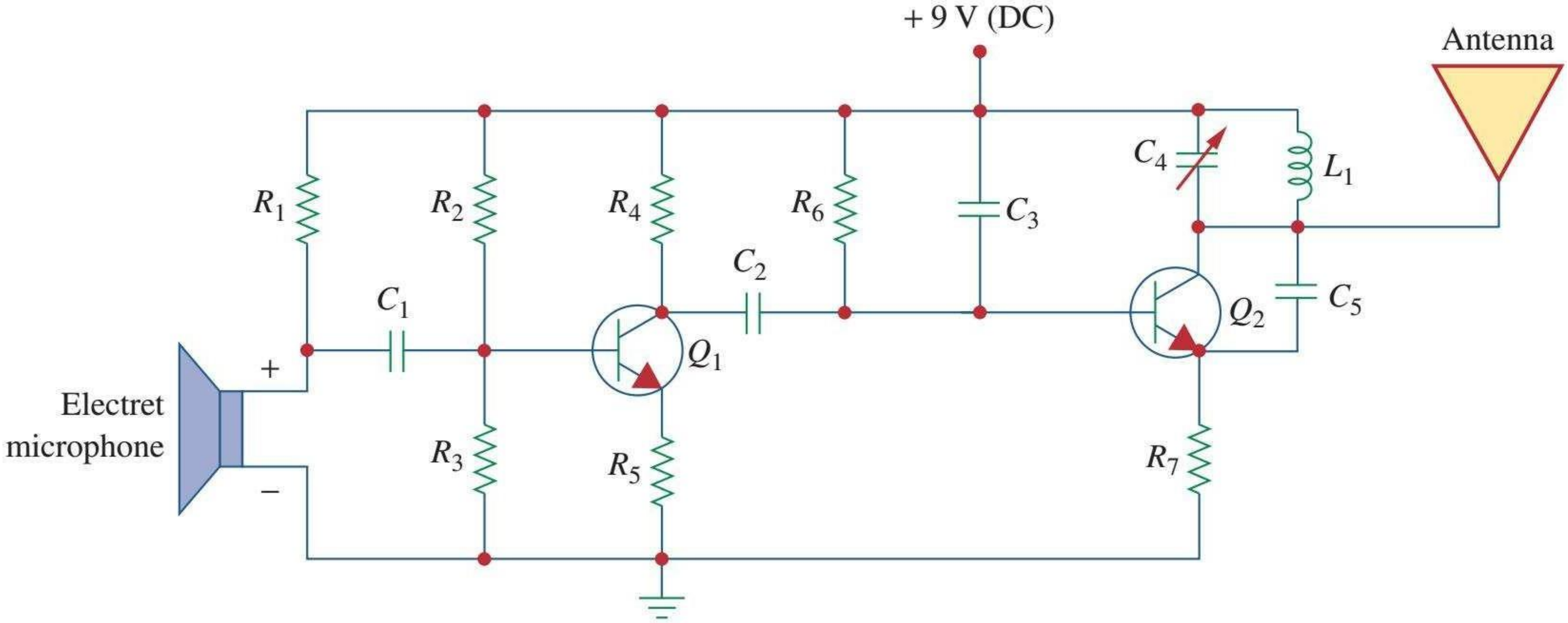
1.1 Introduction: Current and Voltage

- An **electronic circuit** is an interconnection of electrical elements.

Fig 1.1.1: Electric circuit of a radio transmitter



1.1 Introduction: Current and Voltage

- **Charge** is an electrical property of the atomic particles, measured in coulombs (C).
- **Electric current** is the time rate of change of charge, measured in amperes (A).

$$i = \frac{dq}{dt} \begin{cases} i : \text{current} \\ q : \text{charge} \\ t : \text{time} \end{cases}$$

- 1 ampere = 1 coulomb/second.

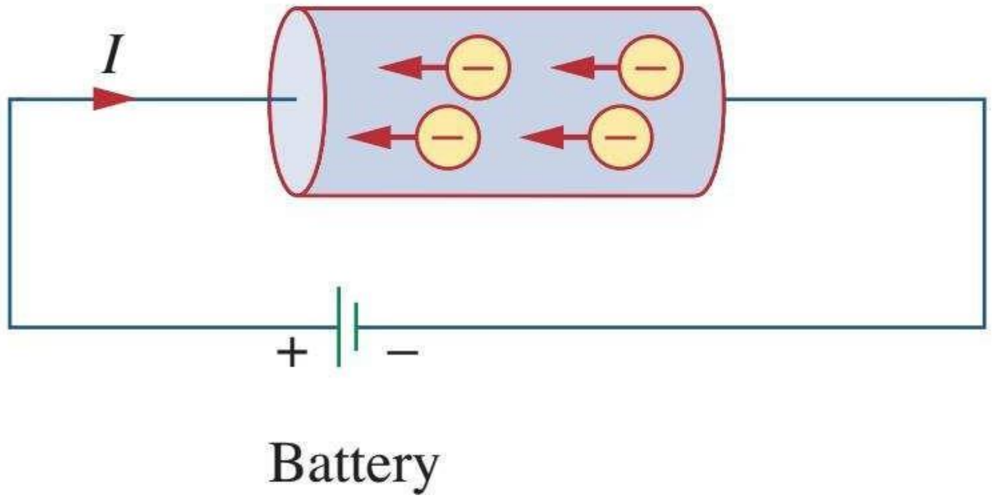


Fig 1.1.2: Electric current due to flow of electronic charge in a conductor

1.1 Introduction: Current and Voltage

- A **<u>direct current (dc)</u>** is a current that remains constant with time.
- An **<u>alternating current (ac)</u>** is a current that varies with time.
- The direction of current flow is the direction of positive charge movement.

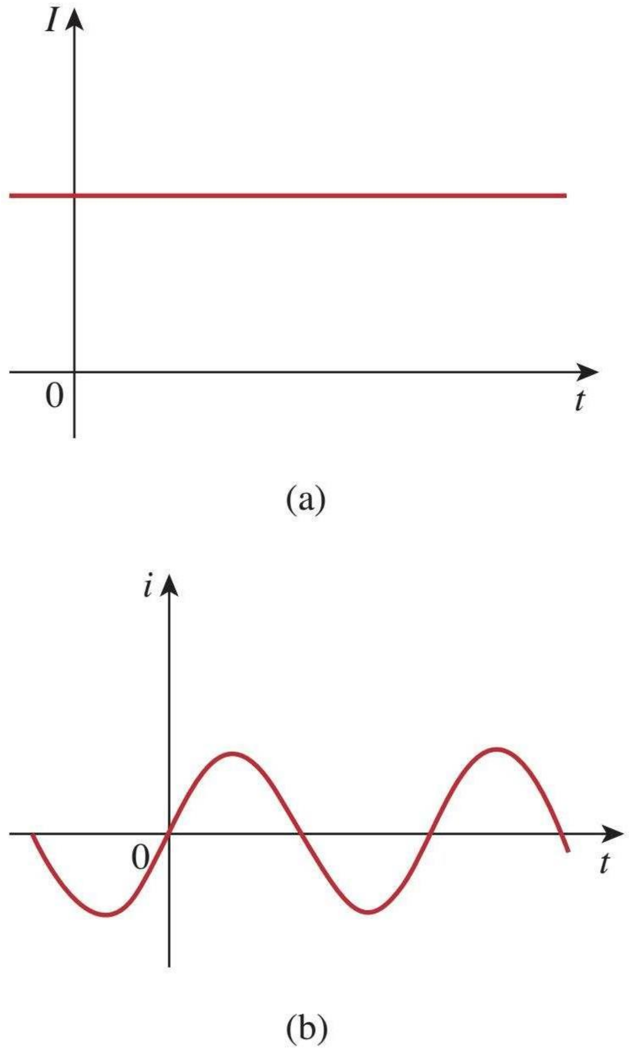


Fig 1.1.3: Two common types of currents: (a) direct current, (b) alternating current

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1.1 Introduction: Current and Voltage

- **<u>Voltage</u>** (or **<u>potential difference</u>**) is the energy required to move a unit charge through an element.
- Voltage is measured in volts (V).

$$u_{ab} = \frac{dw}{dq} \begin{cases} u: voltage\,(V) \\ w: energy\,(joules - J) \\ q: charge\,(C) \end{cases}$$

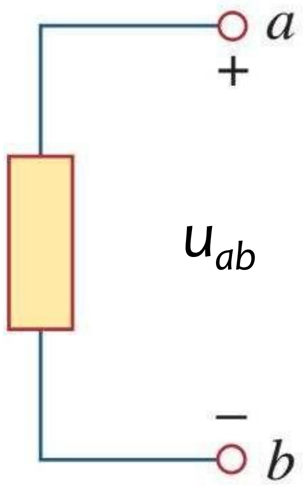

Fig 1.1.4: Polarity of voltage u_{ab}.

- 1 volt = 1 joule/coulomb = 1 Nm/coulomb.

1.1 Introduction: Current and Voltage

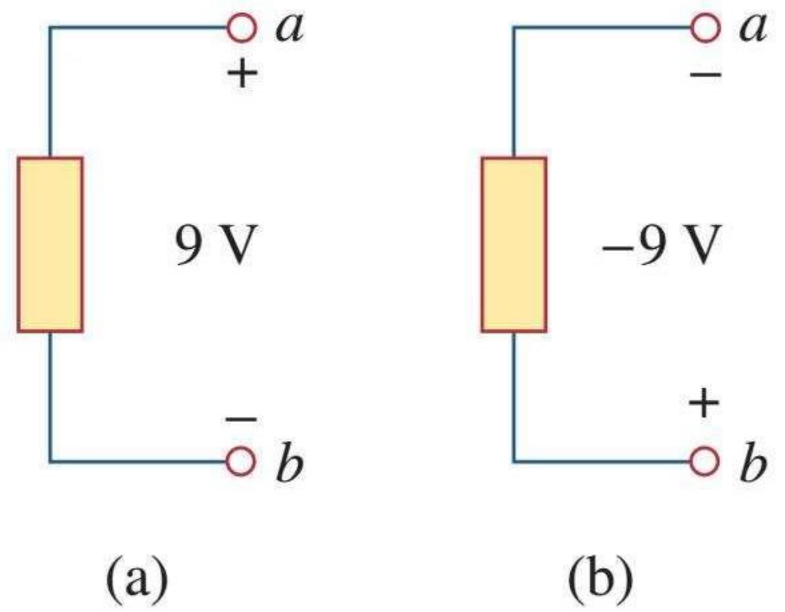

Fig 1.1.5: Two equivalent representations of the same voltage u_{ab}.

- A **dc voltage** is a constant voltage.
- An **ac voltage** is a time-varying voltage.

Concepts and laws

1.1 Introduction: Current and Voltage

➤ **1.2 Circuit Elements**

1.3 Power and Energy

1.4 Laws: Ohm's Law, Kirchhoff's Laws

1.5 Transformations

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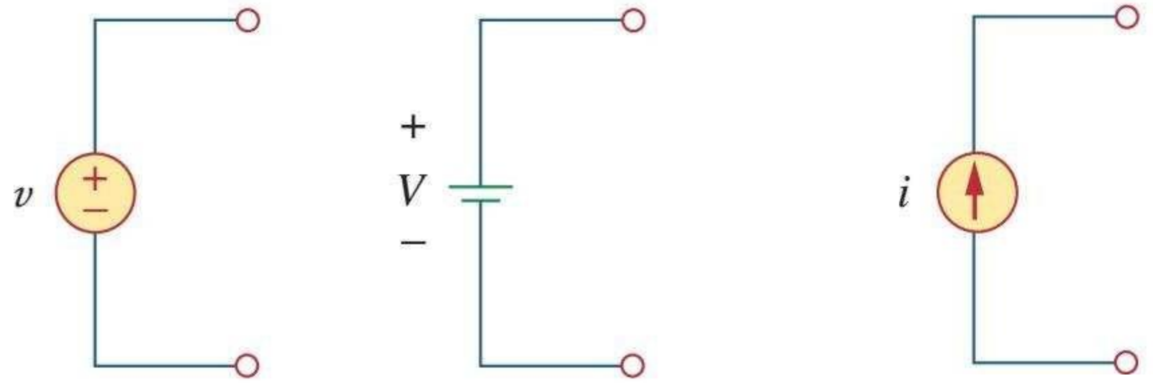
1.2 Circuit Elements

- An **<u>ideal independent source</u>** is an active element that provides a specified voltage or current that is completely independent of other circuit elements.
- There are two kinds of independent sources: **<u>independent voltage source</u>** and **<u>independent current source</u>**.



(a) (b) (c)

Fig 1.2.1: Symbols for independent sources: (a) used for ac and dc voltage; (b) used for dc voltage; (c) used for current source.

1.2 Circuit Elements

- An **ideal dependent** (or **controlled**) **source** is an active element in which the source quantity is controlled by another voltage or current.
- Threre are four possible types of dependent sources, namely:

i. A voltage-controlled voltage source (VCVS).
ii. A current-controlled voltage source (CCVS).
iii. A voltage-controlled current source (VCCS).
iv. A current-controlled current source (CCCS).

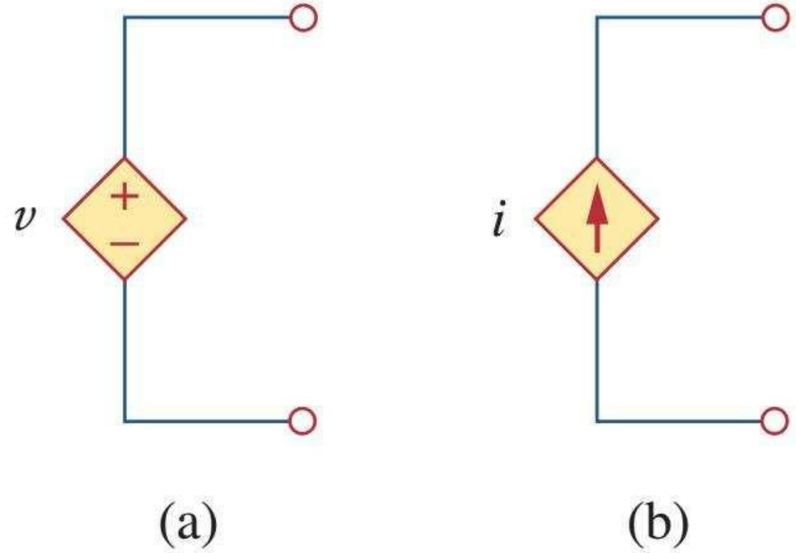
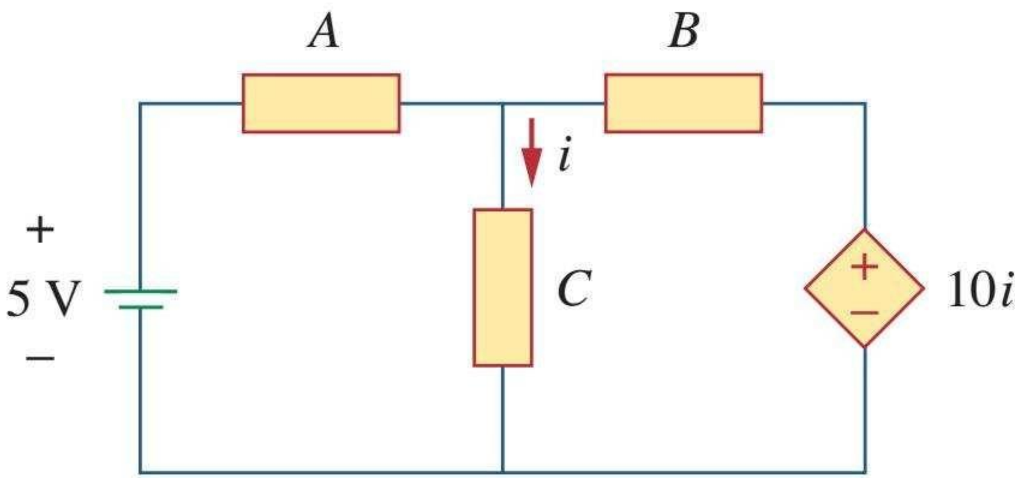
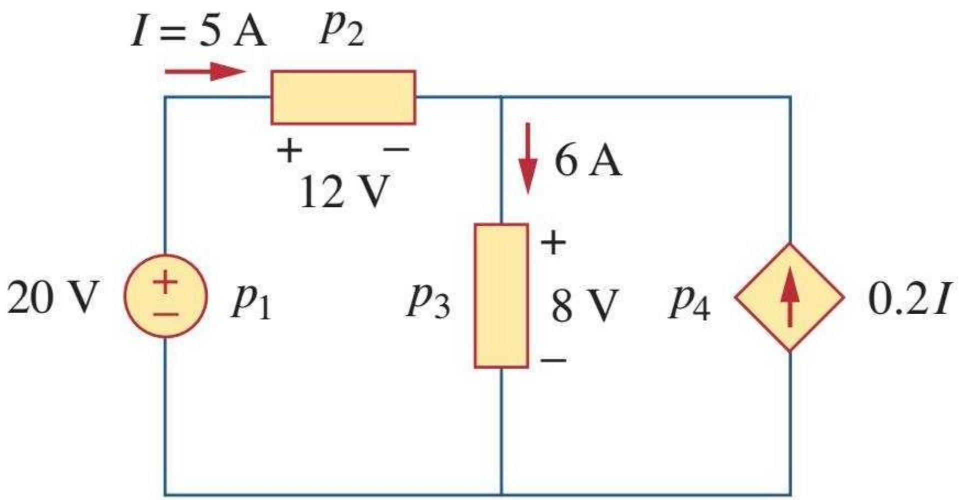


Fig 1.2.2: Symbols for: (a) dependent voltage source, (b) dependent current source.

1.2 Circuit Elements



(a)

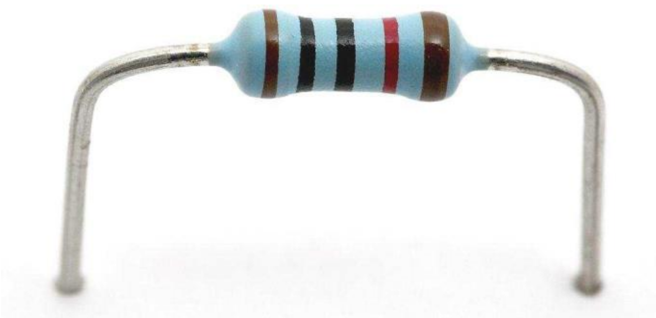


(b)

Fig 1.2.3: Examples of: (a) CCVS, (b) CCCS.

1.2 Circuit Elements

- A **<u>resistor</u>** is a passive electrical component with the primary function to limit the flow of electric current. Resistors are measured in Ohms (Ω).



(a)

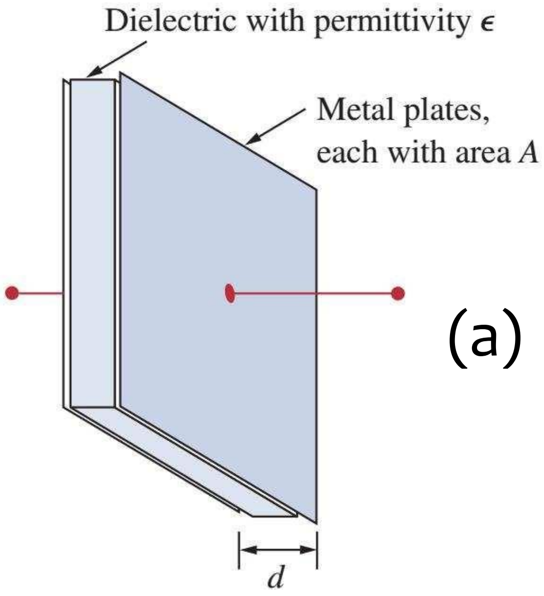
(b)

(c)

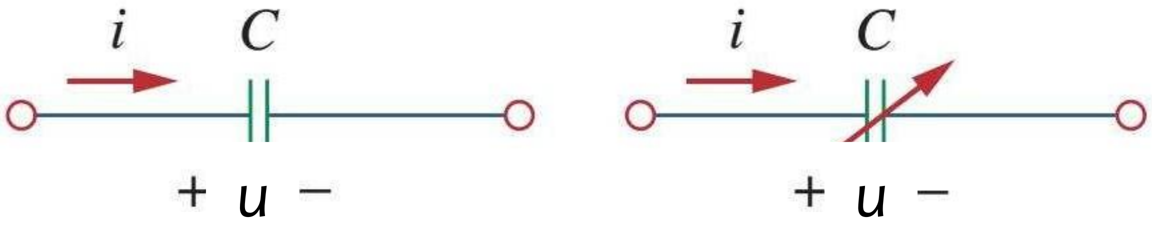
Fig 1.2.4: (a) A common kind of resistor, (b) Resistor symbol - ANSI stadard, (c) another resistor symbol

1.2 Circuit Elements

- A **capacitor** consists of two conducting plates separated by an insulator (or dielectric).
- **Capacitance** is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).



(a)



(b) (c)

1.2.5: (a) A typical capacitor, (b) fixed capacitor symbol, (c) variable capacitor symbol

$$C = \frac{\varepsilon A}{d}; \quad i_C = C\frac{du_C}{dt}$$

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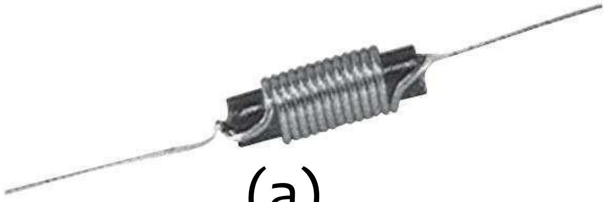
1.2 Circuit Elements

- An **inductor** consists of a coil of conducting wire.
- **Inductance** is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).



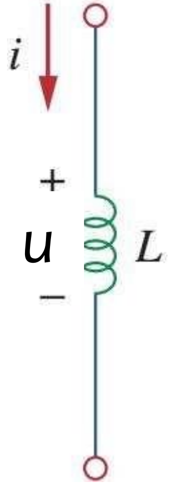


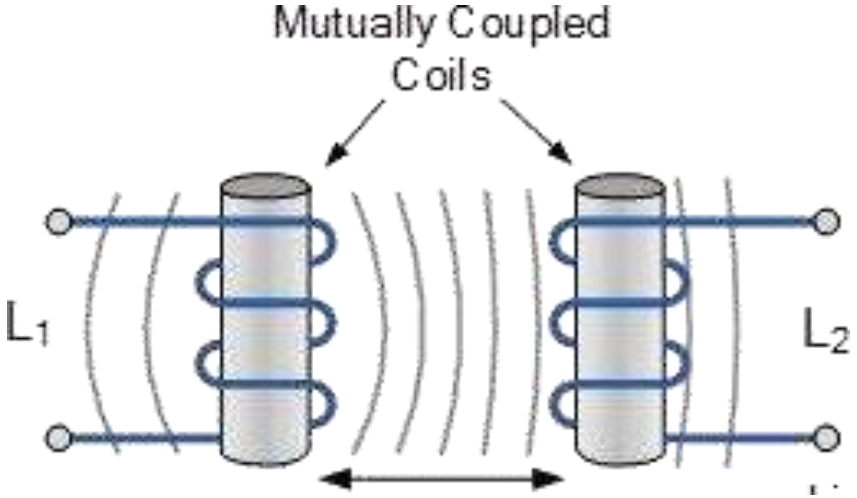
Fig 1.2.6: (a) A typical inductor, (b) fixed inductor symbol

$$L = \frac{N^2 \mu A}{l} \quad \begin{cases} N: \text{number of turns} \\ \mu: \text{permeability of the core} \\ A: \text{cross-sectional area} \\ l: \text{length of the coil} \end{cases}$$

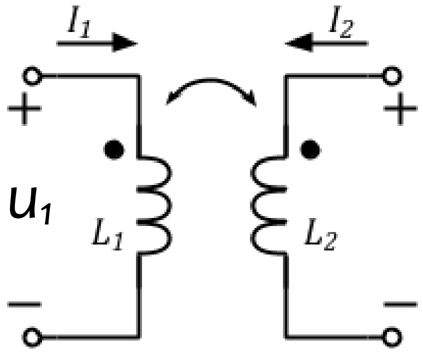
$$u_L = L \frac{di_L}{dt}$$

1.2 Circuit Elements

- When two or more coils are magnetically linked together by a common magnetic flux they are said to have the property of **Mutual Inductance**.



$$M = k_C \sqrt{L_1 L_2} \qquad 0 \le k_C \le 1$$

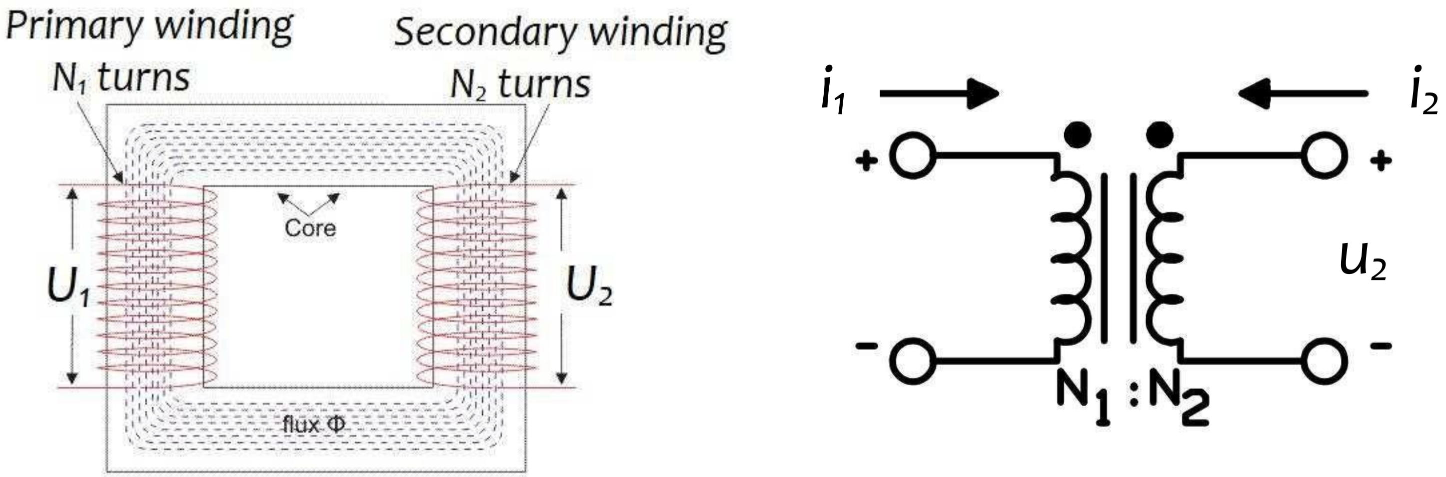


$$\begin{cases} u_1 = L_1 \dfrac{di_1}{dt} + M \dfrac{di_2}{dt} \\ u_2 = L_2 \dfrac{di_2}{dt} + M \dfrac{di_1}{dt} \end{cases}$$

Fig 1.2.7: Mutual inductance and its symbol

1.2 Circuit Elements

- **Electrical transformer** is a static device which transforms electrical energy from one circuit to another without any direct electrical connection, with the help of mutual inductance.
- An **ideal transformer** is an imaginary transformer which does not have any loss in it.



$$\begin{cases} \dfrac{u_1}{u_2} = \dfrac{N_1}{N_2} \\ \dfrac{i_1}{i_2} = -\dfrac{N_2}{N_1} \end{cases}$$

Fig 1.2.8: (Ideal) transformer and its symbol

Concepts and laws

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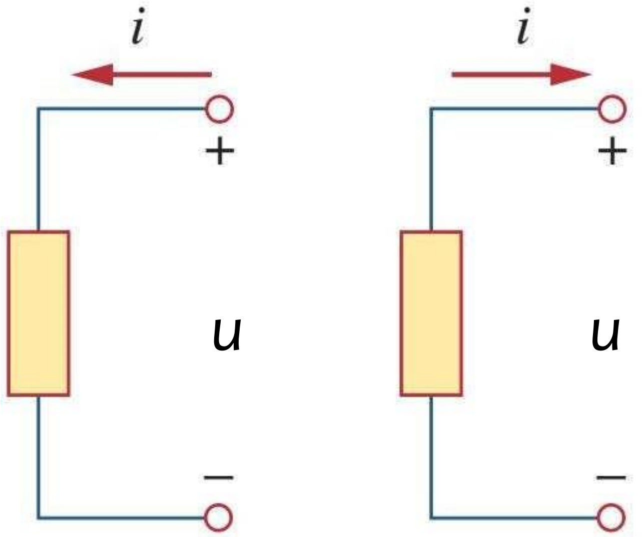
1.3 Power and Energy

- **Power** is the time rate of expending or absorbing energy, measured in watts (W).
- Instantaneous power: $p(t) = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = u(t).i(t)$

$$P = \frac{1}{T}\int_0^T p(t)dt$$

- Average power:
  - *P > 0: absorbing power*
  - *P < 0: supplying power*
- **Law of conservation of energy**:

$$\sum P = 0$$



in a circuit

1.3 Power and Energy

(a) (b)

Fig 1.3.8: (a)
$p = u.i$ (b) $p = -u.i$

1.3 Power and Energy

- **<u>Energy</u>** is the capacity to do work, measured in joules (J).

$$w = \int_{t_0}^{t} p\,dt = \int_{t_0}^{t} u.i\,dt$$

- 1Wh (watt-hours) = 3600J; 1KWh = 3600000J.
- Passive elements (R, L, C, transformer, etc): W > 0.
- Active elements (such as independent voltage or current sources): W < 0.

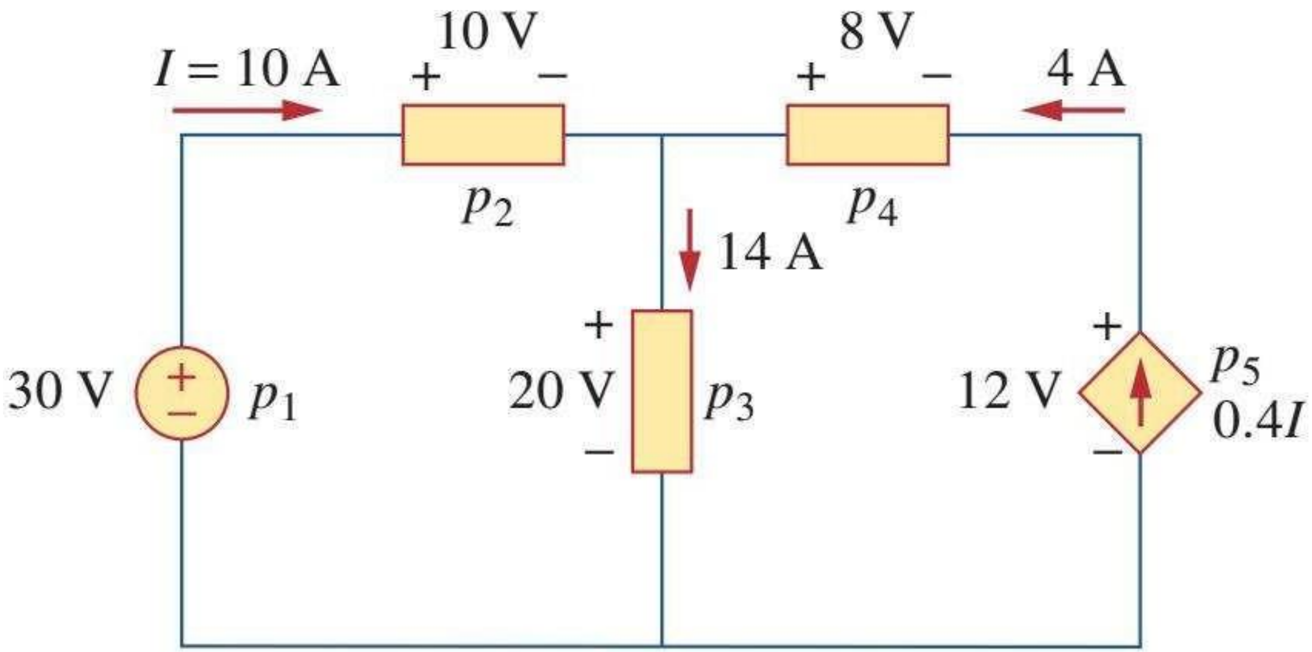
<u>Example 1.3.1:</u> A family in the rural area uses a 40W light bulb in 6 hours per day, the electricity price is 89/KWh. How much does the electricity cost within a month (30 days)?

1.3 Power and Energy

Elements	Average Power	Energy
Resistor	$P_R = Ri^2$	$W_R = R\int_{t_0}^{t} i^2 dt$
Capacitor	$P_C = 0$	$W_C = \frac{1}{2} C u_C^2$
Inductor	$P_L = 0$	$W_L = \frac{1}{2} L i_L^2$

1.3 Power and Energy

- <u>*Example 1.3.2:*</u> Find the power of all elements in the circuit below:



Concepts and laws

1.1 Introduction: Current and Voltage

1.2 Circuit Elements

1.3 Power and Energy

➤ **<u>1.4 Laws: Ohm's Law, Kirchhoff's Laws</u>**

1.5 Transformations

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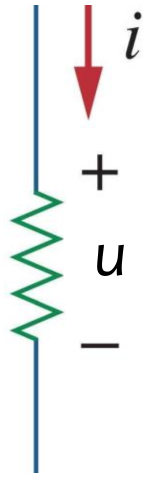
1.4 Laws

- **Ohm's law** states the voltage u across a resistor is directly proportional to the current i flowing through that resistor.

$$u(t) = R.i(t)$$

- A **short circuit** is a circuit element with resistance approaching zero.
- A **open circuit** is a circuit element with resistance approaching infinity.
- Power dissipated by a resistor: $p = u(t).i(t) = i^2(t)R = u^2(t)/R$.
- **Conductance** is the ability of an element to conduct electric

current, it is measured in mhos (℧) or siemens (S).



$$G = \frac{1}{R} = \frac{i}{u}$$

1.4 Laws

- A **branch** represents any two-terminal element.
- A **node** is the point of connection between two or more branches.
- A **loop** is a closed path in a circuit where there is no node which is traversed twice except the initial point.
- A **mesh** is a loop with no path inside.

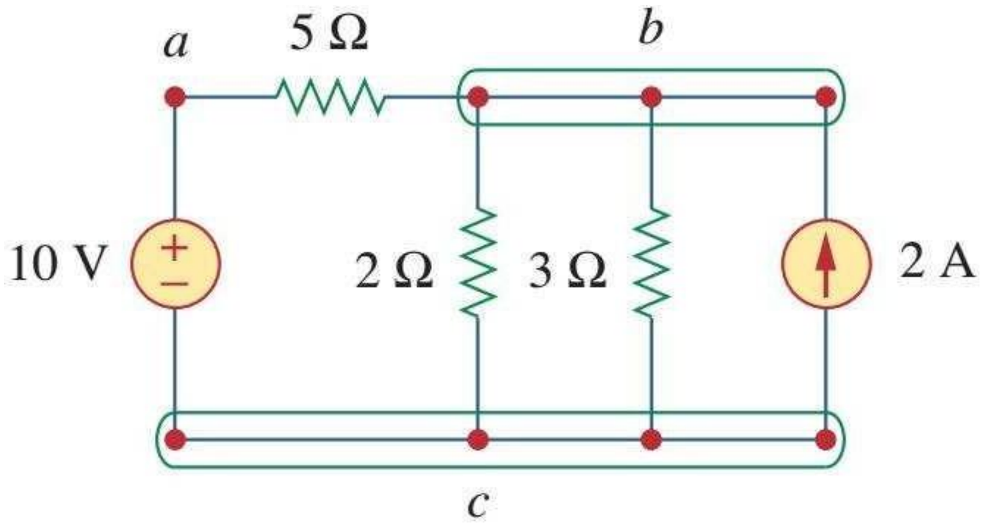
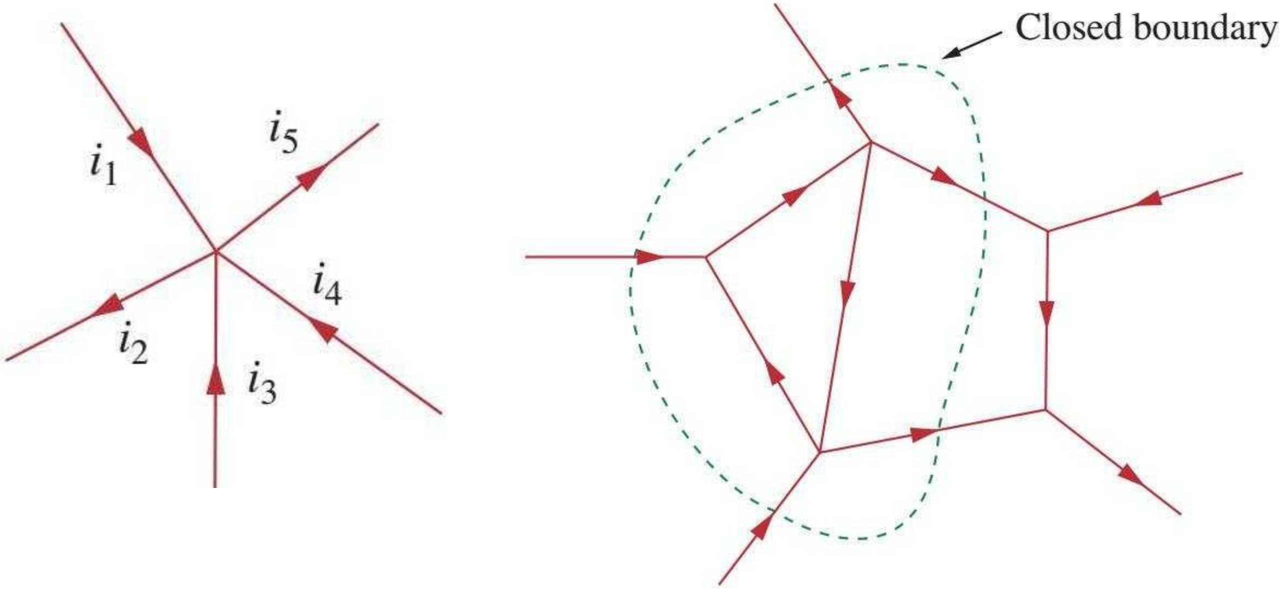


Fig 1.4.1: Branches, nodes, loops and meshes.

1.4 Laws

- **Kirchhoff's current law (KCL)** states that the algebraic sum of currents entering a node (or a closed boundary) is zero.



(a) (b)

Fig 1.4.2: (a) Currents at a node (b) Applying KCL to a closed boundary

For the node in Fig 1.4.2

$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$

or $i_1 + i_3 + i_4 = i_2 + i_5$

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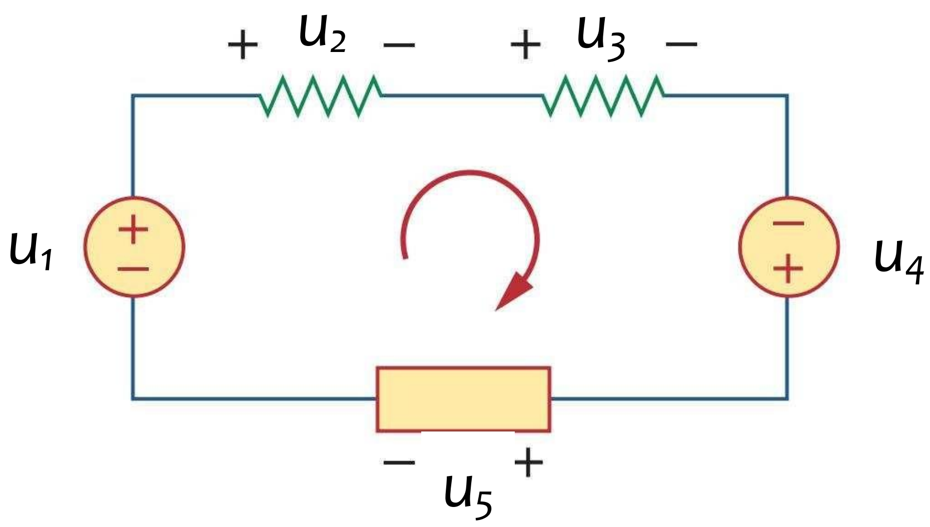
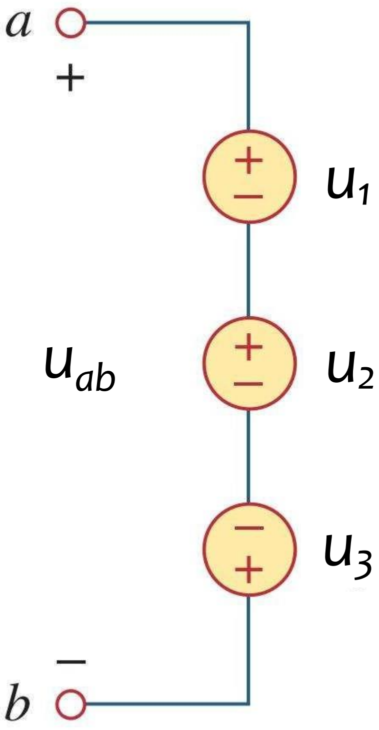
1.4 Laws

- **Kirchhoff's voltage law (KVL)** states that the algebraic sum of all voltages around a loop is zero.



$-u_1 + u_2 + u_3 - u_4 + u_5 = 0$

$\Leftrightarrow u_1 + u_4 = u_2 + u_3 + u_5$



$u_{ab} = u_1 + u_2 - u_3$

Fig 1.4.3: Illustrating KVL

1.4 Laws

- *Example 1.4.1:* Find u_1 and u_2 in the circuit of Fig 1.4.4 (a).
- *Example 1.4.2:* Determine u_x and u_o in the circuit of Fig 1.4.4 (b).

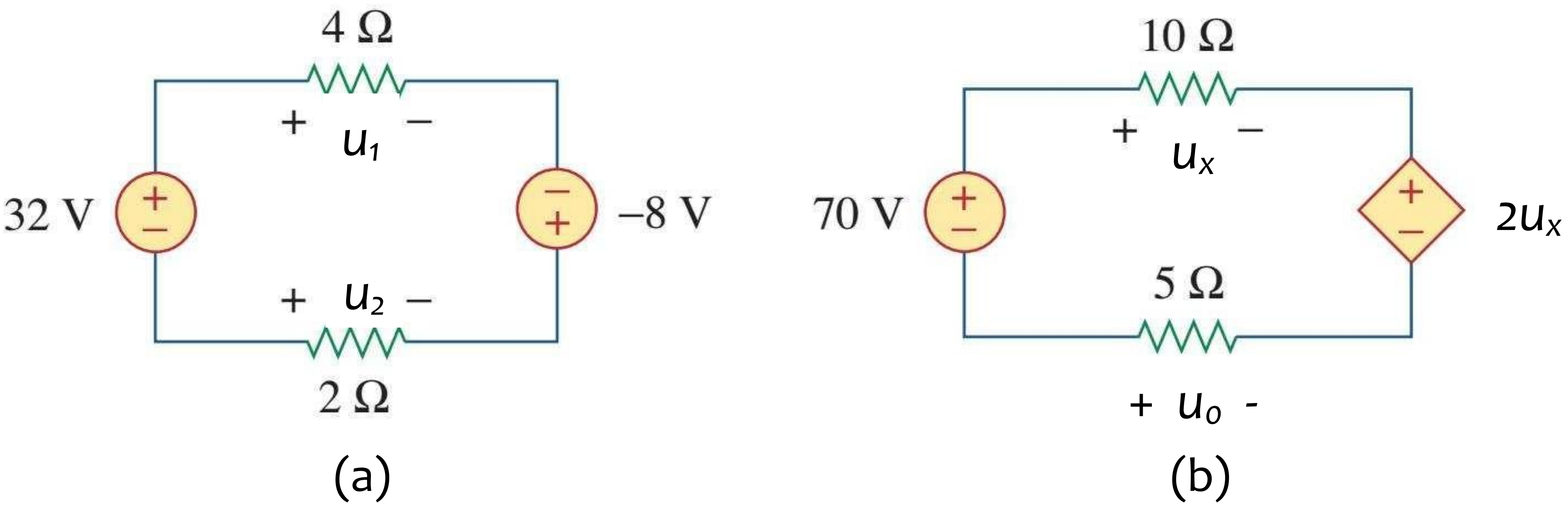


Fig 1.4.4: Circuit for Example 1.4.1 and Example 1.4.2

1.4 Laws

- *Example 1.4.3:* Find i_o and u_o in the circuit of Fig 1.4.5, then calculate the power of all elements.

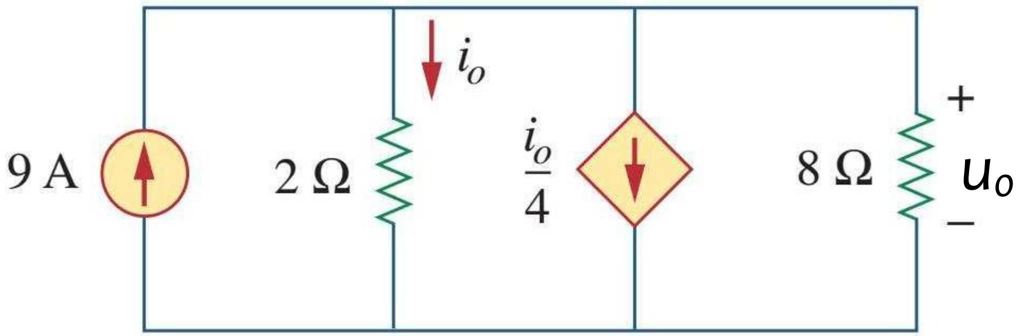


Fig 1.4.5: Circuit for Example 1.4.3

Concepts and laws

1.1 Introduction: Current and Voltage

1.2 Circuit Elements

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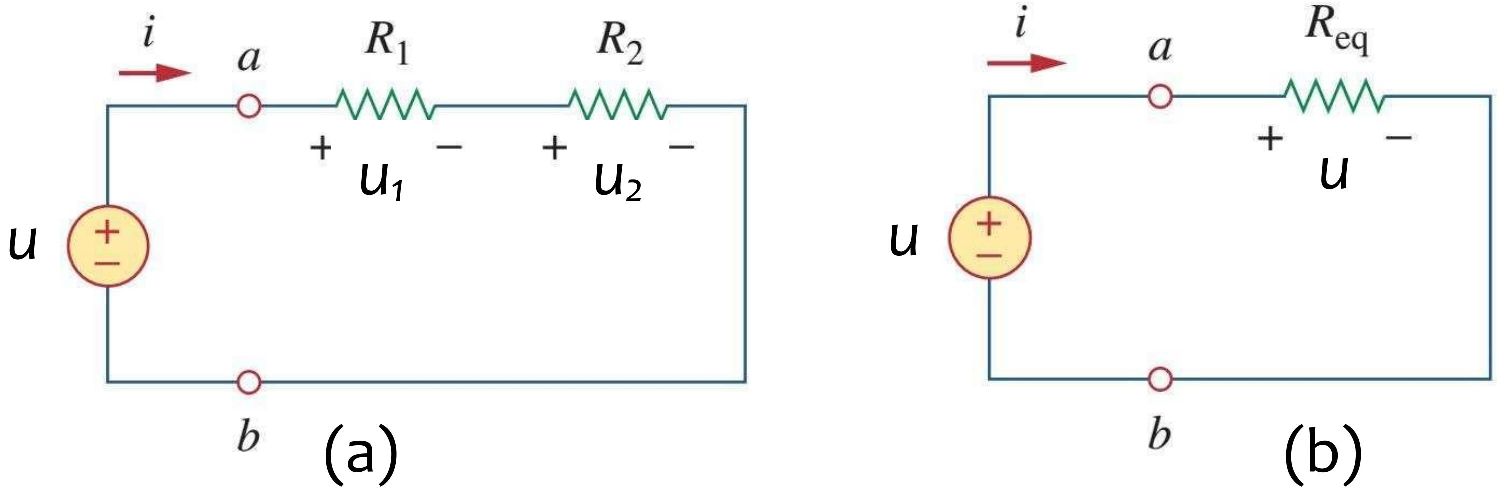
1.4 Laws: Ohm's Law, Kirchhoff's Laws

➤ **1.5 Transformations**

1.5 Transformation

- The **equivalent resistance** of any number of resistors connected in series is the sum of the individual resistances.



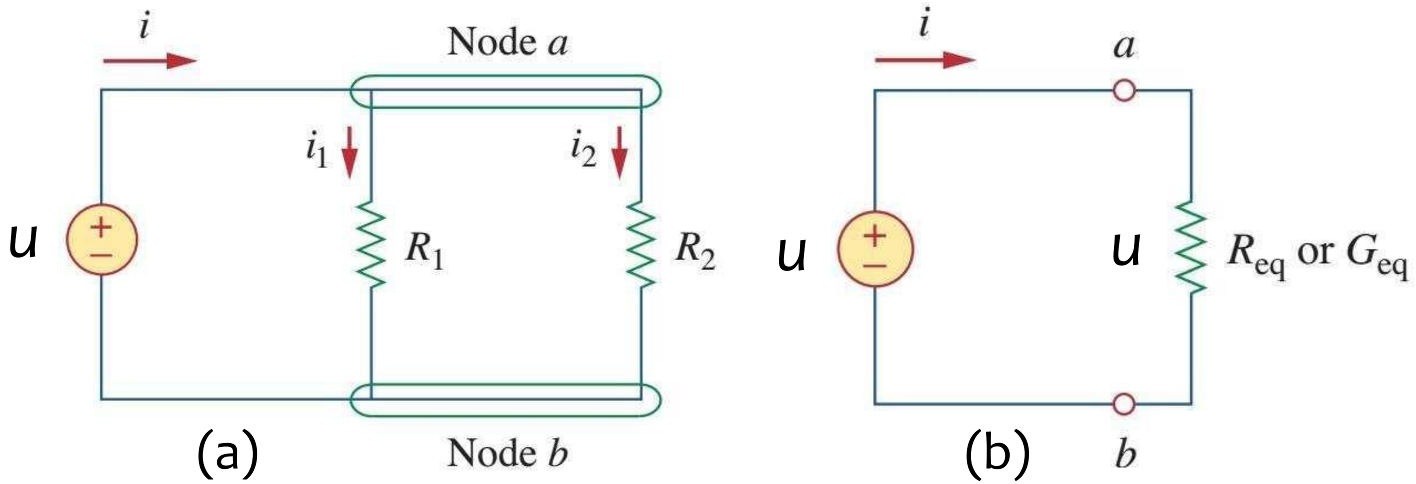
$$R_{eq} = R_1 + R_2$$

Fig 1.5.1: (a) A circuit with two resistors in series and (b) equivalent circuit

- Voltage division: $u_1 = \dfrac{R_1}{R_1 + R_2}u;\quad u_2 = \dfrac{R_2}{R_1 + R_2}u;$

1.5 Transformations

- The **equivalent resistance** of any number of resistors connected in series is the sum of the individual resistances.



$$G_{eq} = G_1 + G_2$$

$$\Leftrightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Fig 1.5.2: (a) A circuit with two resistors in parallel and (b) equivalent circuit

- Current division: $i_1 = \dfrac{R_2}{R_1 + R_2} i; \quad i_2 = \dfrac{R_1}{R_1 + R_2} i;$

1.5 Transformations

- **Wye-Delta (Y-Δ) transformations**:

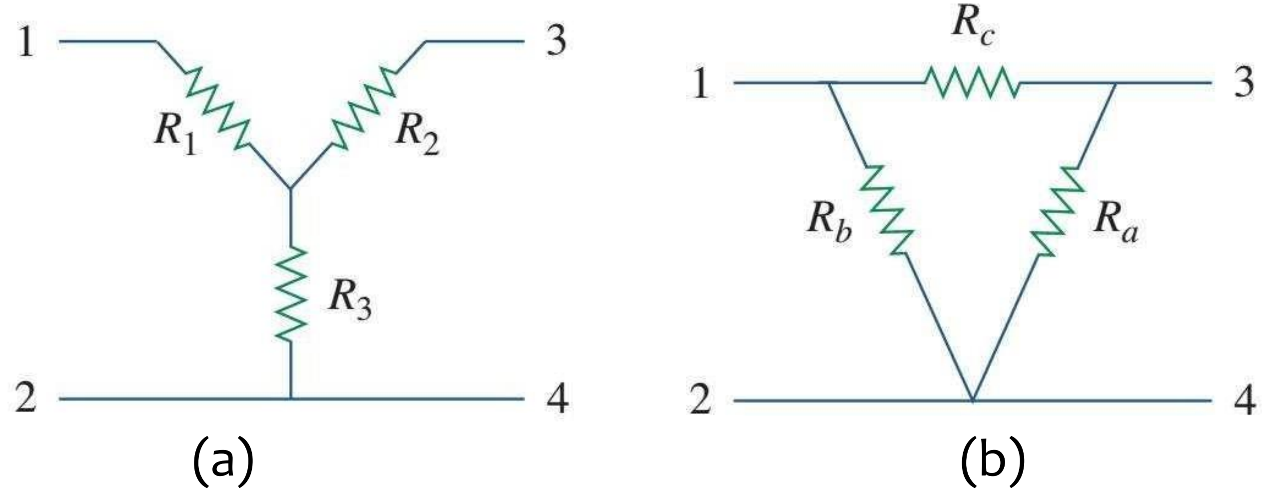


Fig 1.5.3: (a) Y network and (b) Δ network

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \qquad R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

1.5 Transformations

<u>*Example 1.5.1:*</u> For the bridge network in Fig 1.5.4, find R_{ab} and *i*.

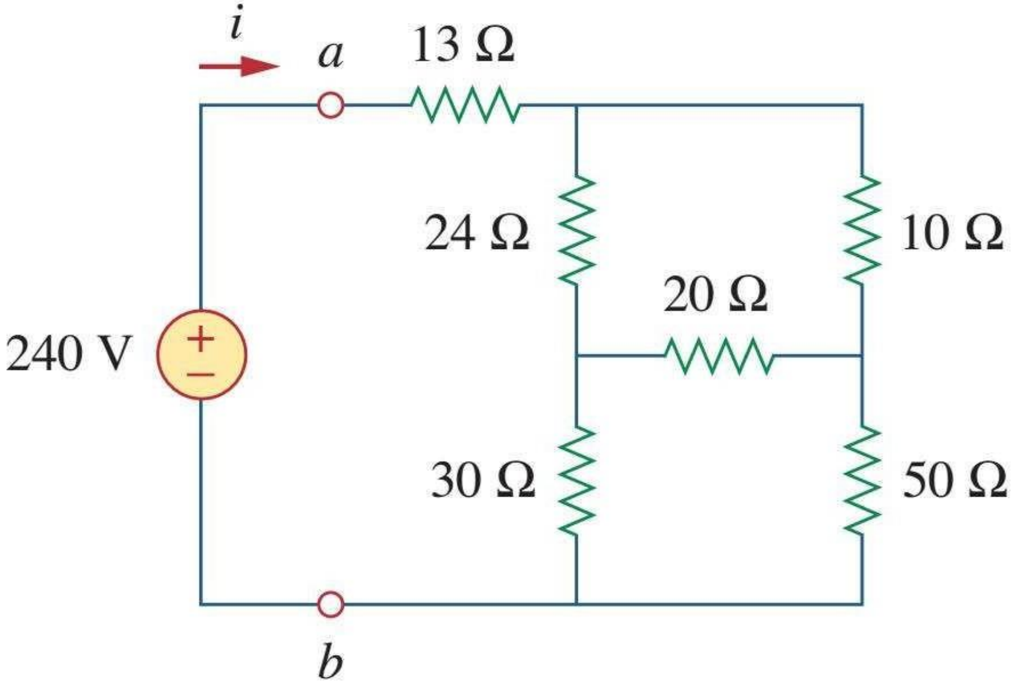


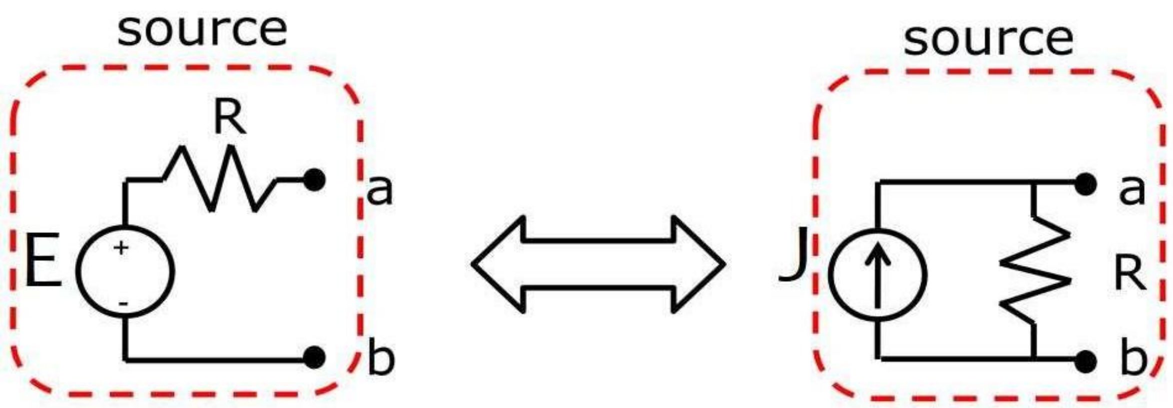
Fig 1.5.4: Circuit for Example 1.5.1

<u>*Answer:*</u> 40Ω, 6A.

1.5 Transformations

- **Source transformations**:

'E' is usually used to denote a voltage source



'J' is usually used to denote a current source

Fig 1.5.5: Source transformations

- Two circuits in Fig 1.5.5 will be equivalent if and only if **E = J.R**.

1.5 Transformations

<u>*Example 1.5.2:*</u> In the circuit shown in Fig 1.5.6, determine u_x and the power absorbed by the 12Ω resistor.

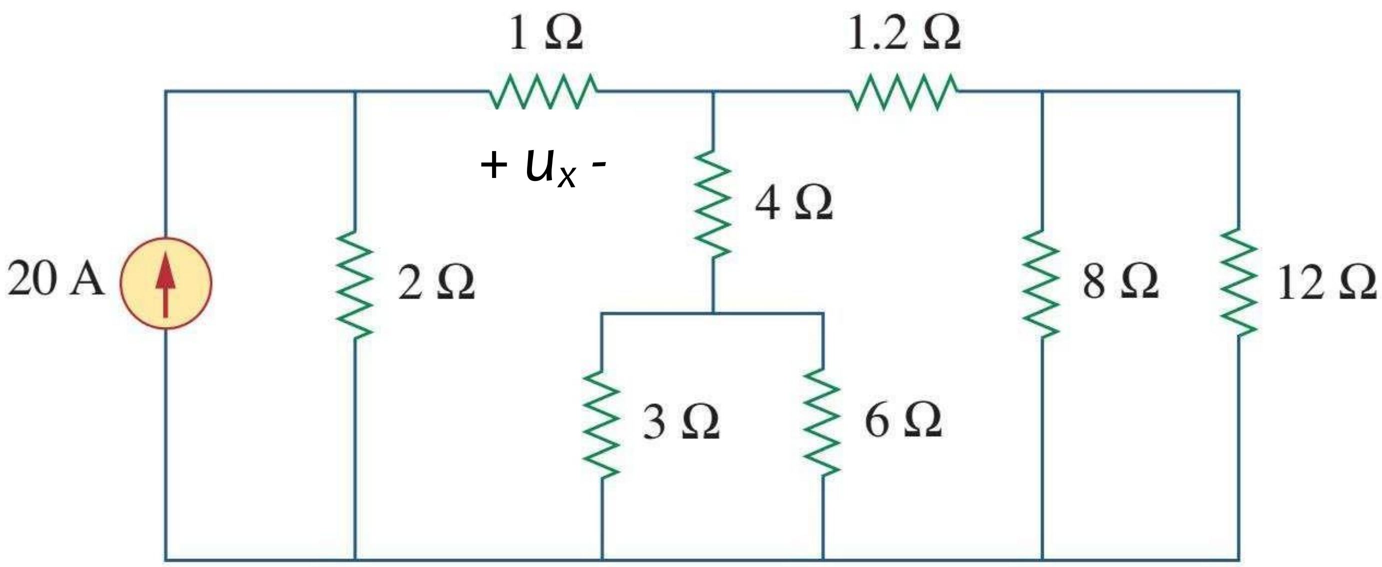


Fig 1.5.6: Circuit for Example 1.5.2

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